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GRAPHS IN GENERAL MATHEMATICS

A DISSERTATION

SUBMITTED TO THE SCHOOL OF GRADUATE STUDIES
IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR
THE DEGREE OF MASTER OF ARTS

COLLEGE OF EDUCATION

BY

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CHAPTER 1

PURPOSE OF THE INVESTIGATION

At present, in Alberta, students enter High School with a grading of H, A, B or C. The average student, in the A and B group, seems to be well provided for in our curriculum; the two extreme groups, (the H and C students) do not seem to be so fortunate.

Generally, it is taken for granted that the "H" student, being, as a general rule, of high intelligence, can look out for himself. The "C" student is the real problem.

The Alberta school curriculum states that the "C" student can not offer any of the regular mathematics courses until he has obtained satisfactory credit in General Mathematics 1. The General Mathematics course must thus serve a double purpose. It must be worthwhile in itself and at the same time it must give the student the proper groundwork for Algebra and Geometry.

Very few students take this course unless they are "C" students. A "C" grading implies a weakness somewhere. Of course we may have a very capable "C" student. The low grading may be due to sickness or too many outside interests. Nevertheless it is a generally accepted fact that the "C" student is usually much below normal in ability to do school work.

Since we live in a democratic country we must try to

give even these weaker students a fair opportunity. The General Mathematics course must be so planned as to meet the needs of these failures in the academic race.

We must not take too much for granted in this course. We will find students who do not know the multiplication tables, who add by some form of counting, who place decimals by guess, and who try to solve most problems by attempting to apply some rule that was memorized while the meaning of that rule is to them what the fourth dimension is to most of us.

What has all this to do with graphs in General Mathematics? It is of fundamental importance; for one must use methods suited to the particular group with which one is working. An explanation of the radio to senior engineering students and to children in the lower grades will naturally be much different. Our treatment of graphs should likewise be different.

The next question which someone may ask is, "Why teach graphs?" Without entering into the age-old question of the value of discipline subjects and transfer of training, I think we answer this question best by saying that we teach graphs because this is one of the best ways to help the student really understand what he is doing. Instead of using abstract symbols he can visualize his problem. He has a chance to really grasp the essentials. It gives him a chance to test his results in an objective manner.

Graphs are not something to be found only in school and in a particular subject, but they are met early in life and they are with us to the grave. The baby expresses how much he wants by a primitive form of graph; the fisherman describes the fish that got away in a like manner. It is a form of expression which is so simple, so understandable, that it can be used to convey messages even to those who understand little or none of our language. It is a universal language in itself.

The essential purpose of this investigation is to take one field of work in General Mathematics and try to discover a suitable program for this particular field. That is, we hope to learn from this study in the one field, methods which will serve as a guide to the problem of successfully treating the entire course.

CHAPTER 11

EARLY TESTS IN GRADES VII TO XII

In an attempt to find out what the various classes of students knew about graphs I gave three short tests. Though the tests were not comprehensive enough to give conclusive evidence of graphical abilities, still, I believe they give us a fairly good indication of these abilities.

The first test consisted of twelve graphical terms which were to be defined. Probably several of these terms should have been deleted and more appropriate terms substituted, but it was only after I had received the answer papers that I realized some of these points.

In test two, I tried to make some of the problems easy enough for all students and some of the problems difficult enough for any student. How well I succeeded is shown clearly by the results.

The third test dealt with interpretation of graphs. To learn how to read a graph and to learn how to construct one, are two different, though related, problems. After receiving the returns I regretted the fact that this test had not been comprehensive enough.

Copies of the tests were given to the students and they were allowed as much time as they needed to finish all the questions. They were asked to answer every question even if they were uncertain as to the correctness of their response.

These tests were given to a fairly large number of students in various classes before the topic of graphs had been taken up this year. The same tests were then given to similar classes later in the year, after the topic of graphs had been taken in the school.

The tests were given to Grades VI, VII, VIII, and IX in this school as well as to General Mathematics students. The records for Grade VI are omitted since they seemed to show that the tests were too difficult for them. I am including VII and VIII records to show the comparison between these grades and higher grades.

The tests were given to village, town and city schools. The various classes tested were Grade IX, Algebra 1, General Mathematics 1, General Mathematics 2, Grade XII Mathematics.

In my own school's records I kept each pupil's score separate and also found the I.Q. of the pupil. The results of the tests when compared with the I.Q. ratings furnish much material for speculation, but since I could not obtain this information from all the other schools I have omitted it in the records given here.

Test 1 - Definitions.

Give a definition of each of the following terms.

Give an example if it will aid you in explaining the term.

- (a) Graph
- (b) Pictogram
- (c) Horizontal Axis
- (d) Broken Diagram or Broken Axis
- (e) Round Number
- (f) Drawing to Scale
- (g) Negative Number
- (h) Average
- (i) Ratio
- (j) Independent Variable
- (k) Dependent Variable
- (l) Inverse Variation

In scoring the test, the definition was considered correct if it showed that the student had a reasonable idea of the meaning of the term. Incorrect and omitted answers were scored separately.

TABLE 1

Results of Grade VII in Test 1.

Definition	Correct	Incorrect	Omitted	Total
(a)	8	1	0	9
(b)	6	2	1	
(c)	7	2	0	
(d)	0	7	2	
(e)	3	4	2	
(f)	5	4	0	
(g)	6	2	1	
(h)	4	5	0	
(i)	1	4	4	
(j)	1	1	7	
(k)	0	2	7	
(l)	0	0	9	

TABLE 11

Results of Grade VIII in Test 1.

Definition	Correct	Incorrect	Omitted	Total
(a)	10	0	0	10
(b)	8	1	1	
(c)	9	1	0	
(d)	0	9	1	
(e)	8	2	0	
(f)	9	1	0	
(g)	10	0	0	
(h)	8	2	0	
(i)	5	4	1	
(j)	1	3	6	
(k)	0	3	7	
(l)	1	2	7	

Comments on Results of Test 1 in Grades VII, and VIII.-

These tests were given in the first part of December before the topic "graphs" was taken up in these grades.

Most of the pupils gave the definition for a broken line graph when required to give the definition of a broken axis or broken diagram. One pupil knew what a broken axis is.

Very few students in grade seven could correctly define "round numbers". Several of the answers indicated that a round number was a whole number or a vulgar fraction, not a mixed number. One pupil in grade eight showed he had a sense of humor at least, for his definition was, "an example of a round number is 0".

The definitions of the terms: Drawing to Scale, Negative Number, and Average were reasonably good. The term "Ratio" proved to be quite difficult. Most of the incorrect answers here were due to the confusion of the term with "radius". The pupils did not know the meanings of the last three items on the test.

TABLE 111

Results of Grade 1X in Test 1.

Definition	Correct	Incorrect	Omitted	Total
(a)	145	13	1	159
(b)	106	15	38	
(c)	152	7	0	
(d)	15	103	41	
(e)	128	27	4	
(f)	148	8	3	
(g)	154	5	0	
(h)	132	26	1	
(i)	41	94	24	
(j)	9	15	135	
(k)	4	19	136	
(l)	8	6	145	

Comments on Grade 1X Test 1.- The only items that seem to call for comment are "d" and "i". In place of the definition of "Broken Axis or Broken Diagram" nearly all of the students gave the definition for a broken line graph. As to the definition of "ratio" I believe some of the answers given will best indicate the type of errors made:

Ratio is the sum which equals two sides.

Ratio is the difference between two numbers.

Ratio is the proportion $2=2$ the ratio is 4.

Ratio is to divide the sum of anything.

Ratio is to find the sum of anything.

Ratio means approximately.

TABLE IV

Results of Algebra 1 in Test 1.

Definition	Correct	Incorrect	Omitted	Total
(a)	93	0	0	93
(b)	85	3	5	
(c)	92	1	0	
(d)	8	26	59	
(e)	78	4	11	
(f)	84	2	7	
(g)	90	1	2	
(h)	86	6	1	
(i)	63	12	18	
(j)	76	8	9	
(k)	72	10	11	
(l)	12	6	75	

Comments on Algebra 1 in Test 1.- Three items on the list proved difficult for the Algebra 1 students. The first was that of the broken diagram. In grade nine many incorrect answers were given for this item, but in Algebra 1 the pupils realized that they did not know the answer, so it was omitted. The few incorrect answers of the Algebra 1 group were much the same as those given by grade nine.

Ratio still proved to be a stumbling block for many students. The chief error was that of saying it represented a sum.

Very few students attempted a definition for inverse variation. Since the meaning of variation was known, as shown by their previous answers, this omission indicated that they failed to understand the term "inverse". This shows a weakness in language work, for they should have noted the likeness to the word "invert", which they probably knew.

TABLE V

Results of Grade XII Mathematics Classes in Test 1.

Definition	Correct	Incorrect	Omitted	Total
(a)	55	0	0	55
(b)	46	0	9	
(c)	55	0	0	
(d)	15	26	14	
(e)	55	0	0	
(f)	55	0	0	
(g)	55	0	0	
(h)	55	0	0	
(i)	52	2	1	
(j)	42	9	4	
(k)	41	10	4	
(l)	38	8	9	

Comments on Grade XII Mathematics Test 1.- As in other classes "d" is confused with a broken line graph. The grade twelve students have learned the meaning of ratio. There is still some difficulty with the last three terms.

Test 11 - Graphical Problems.

1. How many degrees are there in the angular space about the centre of a circle?
2. For most graphs where should the horizontal axis be placed?
3. What is a simple graphical method of comparing two quantities one of which is twice as large as the other?
4. Write 76848 to the nearest thousand.
5. If the numbers in a table range from 0 to 5000, and your paper is ruled in $1/10$ inch squares, you should let 1 inch represent what amount?
6. If the numbers in a table range from 7000 to 7100, explain how you would arrange your units on the axis. Use a sketch.
7. You wish to graph the position of two cars. One goes from A to B and the other from B to A. The car at B starts one hour later than the car at A. Sketch a graph showing A and B and the path which each car takes. (Use any time, speed, and distance you wish.)
8. "It requires 15 minutes plus an additional 15 minutes per pound to roast meat."

Sketch a graph which will show the time required to properly cook any size roast from 1 to 10 pounds.
9. The area of a rectangular chicken yard is to be 1000 sq. ft.

Sketch a graph which will show the corresponding lengths and widths for all reasonable dimensions.

Test 11 (cont.)

10. $A = 3x + 5.$

Draw a graph to represent the above equation.

TABLE VI

Results of Grade VII in Test 11.

Problem	Correct	Incorrect	Omitted	Total
1.	6	1	2	9
2.	5	2	2	
3.	3	3	3	
4.	2	7	0	
5.	3	6	0	
6.	0	9	0	
7.	0	6	3	
8.	0	8	1	
9.	0	9	0	
10.	1	6	2	

TABLE VII

Results of Grade VIII in Test 11.

Problem	Correct	Incorrect	Omitted	Total
1.	9	0	1	10
2.	8	1	1	
3.	9	1	0	
4.	7	3	0	
5.	10	0	0	
6.	0	10	0	
7.	1	4	5	
8.	2	5	3	
9.	3	7	0	
10.	9	1	0	

Comments on Results of Grades VII and VIII in Test 11.-

Except for the first two items the Grade VII class could do little with the test. They attempted most of the problems but showed little knowledge of correct methods.

The Grade VIII class knew how to solve the first five problems. They did well with the last problem. It would seem that their comparative failure on problems 7, 8 and 9 was due to inability to form an equation.

No one knew how to make use of a broken axis, to place the units in problem 6. Most of the students used 10 as their unit, but showed a distance from 0 to 7000 equal to that from 7000 to 7010.

TABLE VIII

Results of Grade IX in Test 11

Problem	Correct	Incorrect	Omitted	Total
1.	103	58	1	162
2.	116	32	14	
3.	87	24	51	
4.	152	10	0	
5.	132	26	4	
6.	78	81	3	
7.	72	42	48	
8.	15	140	7	
9.	2	51	109	
10.	3	94	65	

Comments on Results of Grade 1X on Test 11.- The incorrect answers for the first problem were chiefly 90° , with a few of 180° . Some students said there were no degrees at the centre of a circle.

In problem 2 the chief error was due to confusing horizontal with vertical.

Problem 3 was omitted by a fairly large number of students. It is quite probable that the question was too easy, and the students, not being able to find anything difficult to solve, decided that they couldn't find the catch, so left the problem alone. The incorrect answer given most commonly was, "Use a circle graph."

Problems 4 and 5 were answered fairly well. An answer of 500 or 1000 was taken as correct in the case of problem 5.

Most of the errors in problem 6 were due to taking the interval between 0 and 7000 equal to that between 7000 and 7010. A very common error was that of showing a scale as follows: 0, 1000, 2000, etc.

Problem 7 did not give as much trouble as I had anticipated. Some of the errors were:

Both cars started from the same point.

One car went farther than the other.

Units were given in time and speed rather than time and miles.

In problem 8, very few pupils could decide correctly what to do about the first 15 minutes. The majority ignored

this information; some used a curved line and others a broken line with varying slopes to allow for it.

Only two students correctly solved problem 9. The errors were of various kinds. As shown by the table, a very large number omitted this problem. I classed those problems as omitted when a drawing was made and then crossed out. There were quite a few cases of this crossing out of the attempt. The pupils realized that their graph was incorrect.

The last problem, which I had expected the students to find rather easy, proved to be too difficult for almost everyone. Over half of the incorrect answers were due to the use of $3x+5$ as the unit for one axis. That is, the first number given was $3x+5$, and each succeeding number was a multiple of this as: $6x+10$, $9x+15$ and $12x+20$.

TABLE 1X

Results of Algebra 1 Classes in Test 11.

Problem	Correct	Incorrect	Omitted	Total
1.	126	4	2	132
2.	130	2	0	
3.	115	14	3	
4.	97	35	0	
5.	108	23	1	
6.	123	9	0	
7.	41	70	21	
8.	112	15	5	
9.	38	12	82	
10.	106	16	10	

of 1947.

and the 11 months of 1947, the 11 months of 1948.

Index	1947	1948	1949	1950
100	100	100	100	100
101	101	101	101	101
102	102	102	102	102
103	103	103	103	103
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105	105	105	105	105
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197	197	197	197	197
198	198	198	198	198
199	199	199	199	199
200	200	200	200	200

Comments on Algebra 1 Results in Test 11.- On

problem 7 this class did not do as well as the Grade IX class. The errors were much the same as those made by Grade IX.

The students realized that their attempts were not successful in problem 9 and many crossed out their results. Practically all of these attempts were straight line graphs. In some cases a student would correctly plot the points and then cross out the solution. Several students said, that if the points were correctly plotted, the graph would be a straight line.

TABLE X

Results of Grade XII Mathematics Classes in Test 11.

Problem	Correct	Incorrect	Omitted	Total
1.	53	1	0	54
2.	54	0	0	
3.	52	2	0	
4.	53	1	0	
5.	45	9	0	
6.	52	2	0	
7.	32	20	2	
8.	41	12	1	
9.	13	5	36	
10.	50	3	1	

TABLE I

Properties of the various types of paper used in the tests

Sample	Weight	Thickness	Length	Width
1	100	0.001	100	100
2	100	0.001	100	100
3	100	0.001	100	100
4	100	0.001	100	100
5	100	0.001	100	100
6	100	0.001	100	100
7	100	0.001	100	100
8	100	0.001	100	100
9	100	0.001	100	100
10	100	0.001	100	100

Comments on Results of Grade XII Mathematics Classes

on Test 11.- Problem 7 proved to be quite difficult for them. The errors previously noted for other classes were found here as well. Many of the students made several incorrect attempts before the correct solution was found. It was rather surprising that more of the students who turned in incorrect answers did not realize that the problem was incorrectly done.

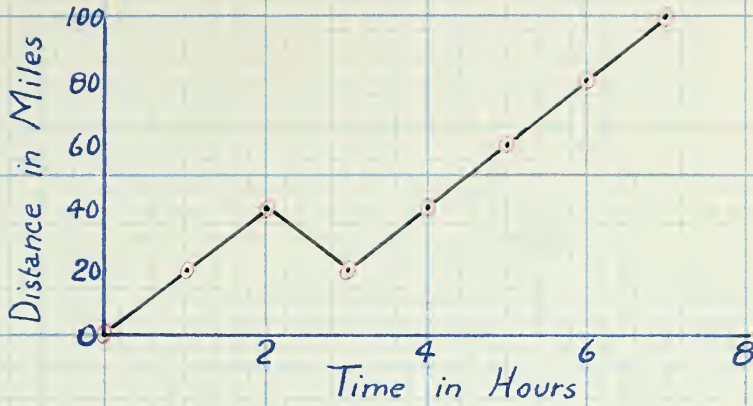
The question of what to do with the first 15 minutes in problem 8, proved to be quite a stumbling block. Most of the students who turned in incorrect answers failed to take account of this factor.

In problem 9, quite a large number started a straight line graph and then crossed it out. As in the Algebra 1 class, some plotted the points correctly, then crossed them out, apparently thinking the graph was incorrect if it were not a straight line.

TEST 3

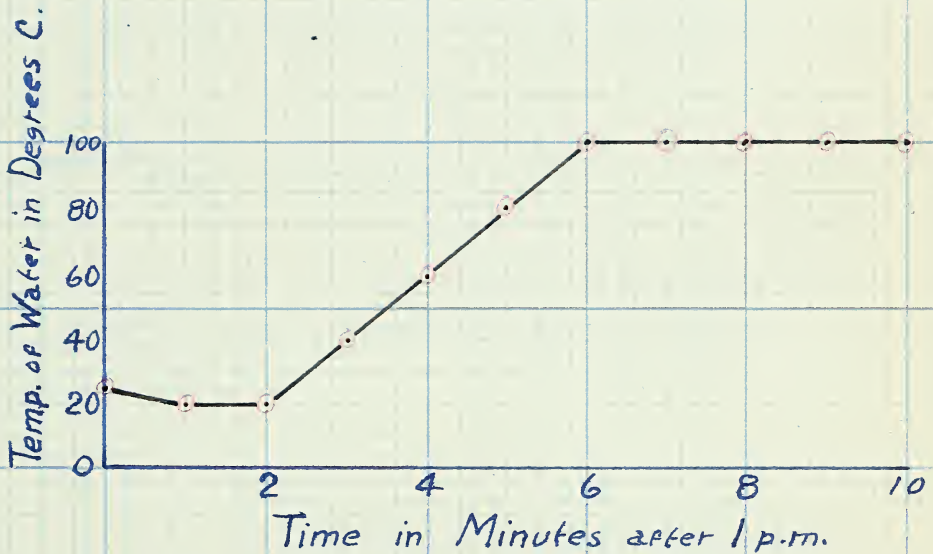
Interpretation of Graphs

1.



Describe fully what this graph tells you.

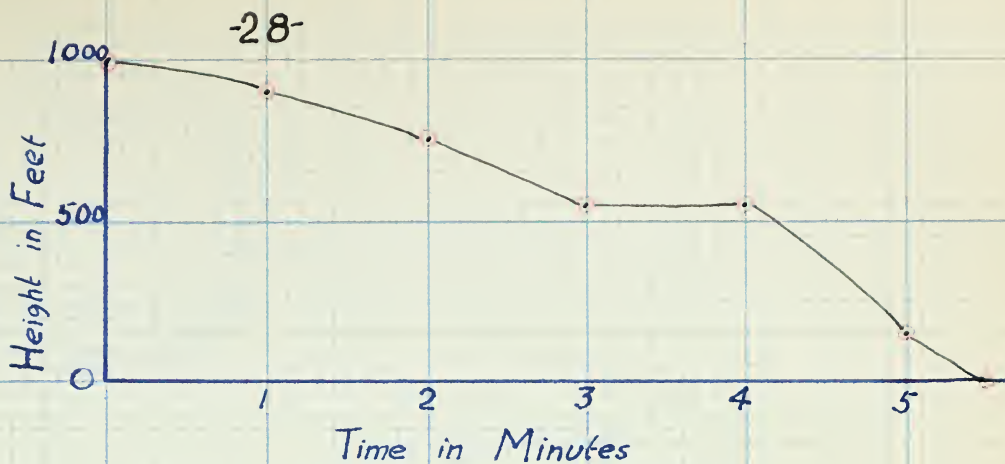
2.



(a) At what time was the heat turned on?

(b) What do the last five points indicate?

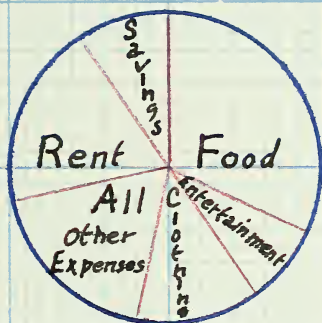
3.



This graph represents the positions of a parachutist who jumped from an aeroplane.

- An error was made in plotting one point. Which is it?
- Why do you think this point is in error?

4.



This graph represents the total expenses of a family for a year.

- Which item makes up the greatest share of the expense?
- Is more spent for clothing than for rent?
- If \$200 is saved each year what amount is spent for food?

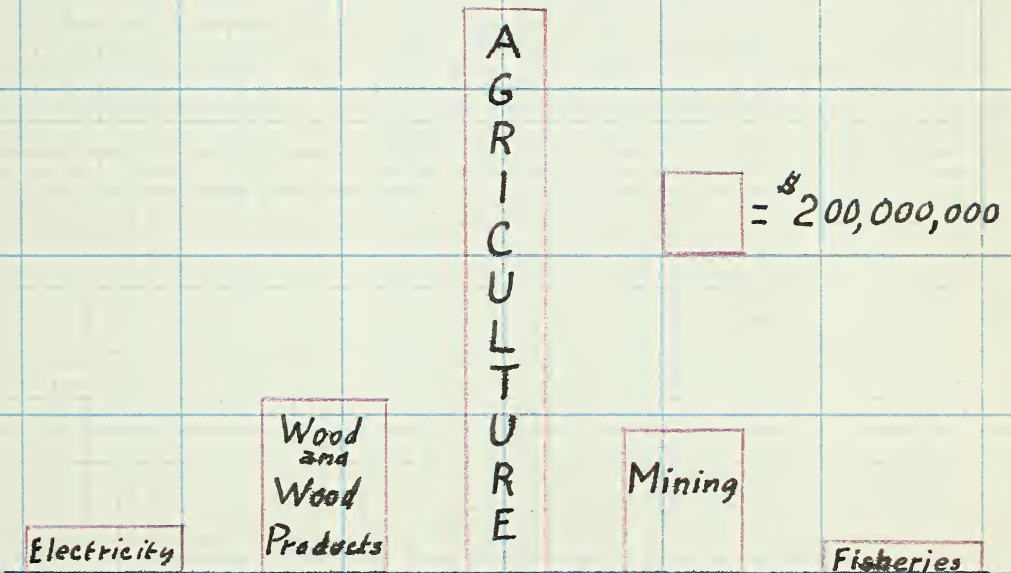
5.



- (a) For the purpose of comparing the various amounts spent, which is the better graph, #4 or #5?
- (b) Give a reason for your answer.

6.

The Chief Industries of Canada



This graph accurately portrays the relative importance of the five chief industries in Canada for a recent year.

Why is it not a good graph?

TABLE XI

Results of Grade VII on Test III.

Problem	Correct	Incorrect	Omitted	Total
1.	0	9	0	9
2(a)	8	1	0	
(b)	1	8	0	
3(a)	7	2	0	
(b)	6	3	0	
4(a)	8	1	0	
(b)	7	2	0	
(c)	2	7	0	
5(a)	8	1	0	
(b)	5	3	1	
6.	1	7	1	

TABLE XII

Results of Grade VIII on Test 111.

Problem	Correct	Incorrect	Omitted	Total
1.	0	10	0	10
2(a)	6	4	0	
(b)	3	6	1	
3(a)	10	0	0	
(b)	9	1	0	
4(a)	10	0	0	
(b)	9	1	0	
(c)	5	5	0	
5(a)	10	0	0	
(b)	7	2	1	
6.	3	4	3	

Comments on Results of Grades VII and VIII on

Test III.- I tried to follow the general practice of starting the test with a relatively easy problem, but the results convinced me that I had failed quite badly. These pupils could not see what had taken place between the second and fourth hours in the first problem.

Most of the pupils failed to give any satisfactory explanation for 2(b). Some acceptable answers were:

The water began to boil.

The heat was turned low.

The temperature would not go higher than 100° .

Water will not get hotter than 100° .

Those who failed to get the correct answer in Problem 3 said that all points should be in a straight line.

Several pupils divided two hundred by three, instead of multiplying, and so obtained an incorrect answer. I accepted as correct any answer between \$500 and \$700.

In problem 6, three pupils in Grade VIII said there was no scale. In Grade VII most of the incorrect answers were: "It is not accurate".

TABLE X111

Results of Grade 1X on Test 111.

Problem	Correct	Incorrect	Omitted	Total
1.	6	154	0	160
2(a)	121	39	0	
(b)	96	64	0	
3(a)	118	40	2	
(b)	110	41	9	
4(a)	158	2	0	
(b)	148	10	2	
(c)	97	56	7	
5(a)	153	7	0	
(b)	134	14	12	
6.	137	17	6	

TABLE II

NUMBER OF STOPS BY TYPE OF VIOLATION

VIOLATION	STOP	VIOLATION	STOP	VIOLATION
1	1	1	1	1
2	1	2	1	(a) 1
3	1	3	1	(b) 1
4	1	4	1	(c) 1
5	1	5	1	(d) 1
6	1	6	1	(e) 1
7	1	7	1	(f) 1
8	1	8	1	(g) 1
9	1	9	1	(h) 1
10	1	10	1	(i) 1
11	1	11	1	(j) 1
12	1	12	1	(k) 1
13	1	13	1	(l) 1
14	1	14	1	(m) 1
15	1	15	1	(n) 1
16	1	16	1	(o) 1
17	1	17	1	(p) 1
18	1	18	1	(q) 1
19	1	19	1	(r) 1
20	1	20	1	(s) 1
21	1	21	1	(t) 1
22	1	22	1	(u) 1
23	1	23	1	(v) 1
24	1	24	1	(w) 1
25	1	25	1	(x) 1
26	1	26	1	(y) 1
27	1	27	1	(z) 1
28	1	28	1	(aa) 1
29	1	29	1	(ab) 1
30	1	30	1	(ac) 1
31	1	31	1	(ad) 1
32	1	32	1	(ae) 1
33	1	33	1	(af) 1
34	1	34	1	(ag) 1
35	1	35	1	(ah) 1
36	1	36	1	(ai) 1
37	1	37	1	(aj) 1
38	1	38	1	(ak) 1
39	1	39	1	(al) 1
40	1	40	1	(am) 1
41	1	41	1	(an) 1
42	1	42	1	(ao) 1
43	1	43	1	(ap) 1
44	1	44	1	(aq) 1
45	1	45	1	(ar) 1
46	1	46	1	(as) 1
47	1	47	1	(at) 1
48	1	48	1	(au) 1
49	1	49	1	(av) 1
50	1	50	1	(aw) 1
51	1	51	1	(ax) 1
52	1	52	1	(ay) 1
53	1	53	1	(az) 1
54	1	54	1	(ba) 1
55	1	55	1	(bb) 1
56	1	56	1	(bc) 1
57	1	57	1	(bd) 1
58	1	58	1	(be) 1
59	1	59	1	(bf) 1
60	1	60	1	(bg) 1
61	1	61	1	(bh) 1
62	1	62	1	(bi) 1
63	1	63	1	(bj) 1
64	1	64	1	(bk) 1
65	1	65	1	(bl) 1
66	1	66	1	(bm) 1
67	1	67	1	(bn) 1
68	1	68	1	(bo) 1
69	1	69	1	(bp) 1
70	1	70	1	(bq) 1
71	1	71	1	(br) 1
72	1	72	1	(bs) 1
73	1	73	1	(bt) 1
74	1	74	1	(bu) 1
75	1	75	1	(bv) 1
76	1	76	1	(bw) 1
77	1	77	1	(bx) 1
78	1	78	1	(by) 1
79	1	79	1	(bz) 1
80	1	80	1	(ca) 1
81	1	81	1	(cb) 1
82	1	82	1	(cc) 1
83	1	83	1	(cd) 1
84	1	84	1	(ce) 1
85	1	85	1	(cf) 1
86	1	86	1	(cg) 1
87	1	87	1	(ch) 1
88	1	88	1	(ci) 1
89	1	89	1	(cj) 1
90	1	90	1	(ck) 1
91	1	91	1	(cl) 1
92	1	92	1	(cm) 1
93	1	93	1	(cn) 1
94	1	94	1	(co) 1
95	1	95	1	(cp) 1
96	1	96	1	(cq) 1
97	1	97	1	(cr) 1
98	1	98	1	(cs) 1
99	1	99	1	(ct) 1
100	1	100	1	(cu) 1

Comments on Results of Grade IX on Test III.- As in Grades VII and VIII the pupils do not know what has happened between the second and third hours in problem one. Five pupils had the following answer:

In 1 hour he went 20 miles.

" 2 hours " " 40 " .

" 3 " " " 60 " .

" 4 " " " 100 " .

" 5 " " " 160 " .

" 6 " " " 180 " .

I have tried to find some explanation for this answer, but the only one I can see is that one pupil put down these figures and the others copied them. Upon checking these papers I found that two of them had exactly the same error on problem 3 and two of them omitted this problem. The other problems were correct.

Most of those who were in error on problem 2(b) said that the heat was taken away. They failed to observe that the temperature remained constant for four minutes. If you took the heat away the temperature would fall. Eight pupils gave 1.03 as the answer for 2(a).

In problem 3 two pupils pointed out an error on my part. These pupils said that the parachutist would fall farther during the first minute since the parachute would not open immediately. Most of the errors were made by saying that the point should be on 6 not between 5 and 6.

The common error in problem 4(c) was that of dividing \$200 by 3, thus obtaining \$67 for the answer. Six pupils gave \$6 for the amount spent for food - an error in placing the decimal.

The grade nine pupils found little difficulty in showing why the graph in problem 6 was not a good one.

TABLE XIV

Results of Algebra 1 on Test 111.

Problem	Correct	Incorrect	Omitted	Total
1.	16	115	0	131
2(a)	104	27	0	
(b)	107	22	2	
3(a)	124	6	1	
(b)	118	8	5	
4(a)	131	0	0	
(b)	131	0	0	
(c)	105	21	5	
5(a)	130	1	0	
(b)	128	0	3	
6.	129	1	1	

Comments on Results of Algebra 1 in Test 111.-

Problem 1 is still found to be very difficult. Comments made for the other grades seem to apply here as well.

In problem 2(a) a fairly large number of students failed to read the horizontal scale correctly. The time was given as 2 o'clock.

Of the errors in problem 4(c), all but five of the students made the mistake of dividing by three to get their answer. Three students said there was no way of telling how much was spent for food.

TABLE XV

Results of Grade XII Mathematics Classes on Test III.

Problem	Correct	Incorrect	Omitted	Total
1.	38	16	0	54
2(a)	43	11	0	
(b)	48	6	0	
3(a)	47	7	0	
(b)	47	7	0	
4(a)	54	0	0	
(b)	54	0	0	
(c)	36	18	0	
5(a)	52	2	0	
(b)	50	2	2	
6.	51	3	0	

Comments on Results of Grade XII Mathematics Classes

on Test 111.- Many of these pupils found difficulty with problem 1. Two of them gave the same answer as the five previously mentioned in the Grade IX results. Since these two were not in the same school as those in Grade IX, these similar incorrect answers were not due to copying.

In problem 2(a) most of the errors were due to taking the numbers on the horizontal axis as representing hours, not minutes.

In problem 3, four pupils said, "The horizontal lines should equal the vertical lines; therefore the last two points are incorrect."

Most of the errors in problem 4(c) were the same as those for the other classes - they divided by three instead of multiplying. Four students placed the decimal incorrectly.

General Comments on Results of Tests.- All classes seem to know very little about a broken axis or a broken diagram. Very few could give a definition of the term; and on the second test, where this device could have been used, only a few made use of it.

In test two, problems 7, 8 and 9 proved too difficult for most students. I believe these problems should not have been so poorly done. These results seem to prove that, while many can read certain graphs and make others, still, they fail to really understand them.

Results of all grades on 4(c), test 3, shows us that

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many pupils use some rule to get an answer; but they do not check the answer to see if it is reasonable. I believe that this point presents a real problem for the mathematics teacher.

CHAPTER 111

PRELIMINARY TESTS IN GENERAL MATHEMATICS CLASSES.

All of the pupils who were in General Mathematics 1 were "C" students. The pupils who took the tests were from various types of schools: city, town and village. The tests were given early in the year before the topic "graphs" had been taken up.

In the case of General Mathematics 2 students, some were "C" students and some were those who had failed to obtain a "B" standing in Algebra 1. There were a few students in General Mathematics 2 who had satisfactorily passed Algebra 1, but who did not wish to take Grade XII Mathematics.

The tests were the same as those listed in Chapter 11. The instructions and marking were the same as for the other classes.

The average I.Q. of my General Mathematics 1 pupils was 93. I would like to have obtained the I.Q. ratings of all the General Mathematics students in other schools but I could not get these, so I have not recorded mine individually. My reason for not using the I.Q. ratings of my own classes as a basis for various arguments is that the total number of students is too low to give reliable results.

I think it will be readily agreed that the pupils in the General Mathematics classes are of fairly low intelligence as compared to those of other classes in the High School. The opinions of the teachers who teach these classes seem to be quite uniform in this respect. When I asked one teacher

if he would give the tests to his pupils he replied, "I'll give them if you wish; but I can tell you, just as well without giving them, what the results will be."

TABLE XV/

Results of General Mathematics 1 on Test 1.

Problem	Correct	Incorrect	Omitted	Total
(a)	81	3	2	86
(b)	10	18	58	
(c)	48	31	7	
(d)	0	65	21	
(e)	51	33	2	
(f)	83	2	1	
(g)	36	40	10	
(h)	84	2	0	
(i)	14	61	11	
(j)	5	43	38	
(k)	3	36	47	
(l)	1	24	61	

TABLE 1

Summary of the results of the tests of the

Test	Value	Location	Time	Notes
1	10	10	10	(1)
2	20	20	20	(2)
3	30	30	30	(3)
4	40	40	40	(4)
5	50	50	50	(5)
6	60	60	60	(6)
7	70	70	70	(7)
8	80	80	80	(8)
9	90	90	90	(9)
10	100	100	100	(10)
11	110	110	110	(11)
12	120	120	120	(12)
13	130	130	130	(13)
14	140	140	140	(14)
15	150	150	150	(15)

Comments on Results of General Mathematics 1 classes

on Test 1.- Practically all the students know what a graph is, and they can properly define it. Not many know what a pictogram is, and few of them tried to define it.

Only 58% of the students tested could define or illustrate the term "horizontal axis". Sixteen gave the definition of a vertical axis and eight said the horizontal axis was "lines running across the page". In three cases of the latter, illustrations were given of bar graphs running horizontally.

As in other classes, most of the attempts to define a broken axis or broken diagram resulted in a definition of a broken line graph.

The majority of mistakes in defining, "a round number" were made because students gave it the meaning of "approximately" and then showed that there was no effort made to simplify the number. As an example of this, some gave 7 as the round number for 6. Another common answer given by these students is "any number divisible by 2 is a round number."

Drawing to scale was well understood by this group. The most frequent example given was, "let one inch represent 1 mile."

To illustrate the mistakes made in defining a negative number, I give here some of the answers:

"A negative number is one number which cannot be divided by another number."

"A negative number is the minus sign, e.g. 5-8. -8 can be changed to positive by adding."

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"A negative number is one in which no two numbers when multiplied together will divide. An example of this is 19."

"A negative number is the answer to a subtraction question."

"A negative number is a number which can be divided by two, such as 6, 8, 10."

I conclude that this problem gives us a clue to General Mathematics students' troubles. They do not understand the term but know, from experience, that they are expected to do something. They get a hint from one example and then apply this to every case.

The following are some answers given as definitions of "ratio":

"Ratio is part of a circle."

"Ratio is a short way to show how a problem is done. The ratio of a circle is $2r$."

"Ratio is one number after another - 5, 6, 7, 8."

"Ratio is what one thing is to another. For example 1:2 equals 9:10."

"Ratio is what one number is to another as in

$$\frac{10}{15} = \frac{9}{x} = \frac{9}{27}."$$

The students in general did not know the meaning of the last three terms. One student said, "A dependent variable is one you can depend on to change and an independent variable is one that might change but probably won't".

the first of these is the fact that the

the second is the fact that the

the third

the fourth is the fact that the

the fifth

the sixth is the fact that the

the seventh

the eighth is the fact that the

the ninth is the fact that the

the tenth is the fact that the

the eleventh is the fact that the

the twelfth

the thirteenth is the fact that the

the fourteenth

the fifteenth

the sixteenth is the fact that the

the seventeenth

the eighteenth is the fact that the

the nineteenth is the fact that the

the twentieth

the twenty-first is the fact that the

the twenty-second

the twenty-third is the fact that the

the twenty-fourth is the fact that the

the twenty-fifth is the fact that the

the twenty-sixth

TABLE XY11

Results of General Mathematics 11 on Test 1.

Problem	Correct	Incorrect	Omitted	Total
(a)	26	0	1	27
(b)	24	1	2	
(c)	23	4	0	
(d)	4	18	5	
(e)	24	2	1	
(f)	27	0	0	
(g)	26	1	0	
(h)	26	1	0	
(i)	20	5	2	
(j)	1	4	22	
(k)	1	4	22	
(l)	0	1	26	

Comments on Results of General Mathematics 2 Class

on Test 1.- As in all other classes very few know the meaning of a broken axis. The last three terms are not known but all the other terms are known quite well.

An interesting case was that of a student who said that the average was the middle term of a group of numbers, and who then gave the following as an example: "7, 6, 1, 3, 10; the average is 1."

The pupil who incorrectly defined a negative number said, "A negative number is an odd number."

Another interesting point was the fact that when the General Mathematics 2 students did not know the answer, it was omitted, as a general rule; whereas in General Mathematics 1, some answer was given. Examples (j), (k), and (l), illustrate this quite well.

TABLE XVII

Results of General Mathematics 1 on Test 11.

Problem	Correct	Incorrect	Omitted	Total
1.	80	4	1	85
2.	49	32	4	
3.	72	5	8	
4.	40	45	0	
5.	23	60	2	
6.	41	42	2	
7.	7	74	4	
8.	2	71	12	
9.	0	46	39	
10.	1	22	62	

TABLE IV

Summary of results of the 1954-55 season

Period	Area	Yield	Yield	Yield
1	1	1	1	1
2	2	2	2	2
3	3	3	3	3
4	4	4	4	4
5	5	5	5	5
6	6	6	6	6
7	7	7	7	7
8	8	8	8	8
9	9	9	9	9
10	10	10	10	10
11	11	11	11	11
12	12	12	12	12

Comments on Results of General Mathematics 1 on

Test 11.- Errors on problem 2 were chiefly due to the confusion of the term "horizontal axis" with the term "vertical axis".

In problem 4 the incorrect answers, as well as the number who made each error are given below:

20 gave	80000
10 gave	8000
8 gave	76848.000
3 gave	10000
3 gave	76000
2 gave	100000

It is fairly evident how each incorrect answer has been obtained.

Following a rule thoughtlessly is well illustrated in problem 5. A fairly large number of students made errors of this type -- "Let the page be 9 inches. Then $1" = \frac{5000}{9} = 555 \frac{5}{9}."$

Another common error was " $1" = 10."$ In several cases, this was explained by saying that, since the paper was ruled in $\frac{1}{10}$ - inch squares, 1 inch must equal 10.

As in other classes, the error most commonly made in problem 6, was in taking equal intervals for 0 to 7000 and 7000 to 7010. Ten students used the following scale:

0, 1000, 2000, 8000.

Problem 7 proved to be very difficult. A great number of different types of error were found.

Nearly everyone neglected the initial 15 minutes in problem 8. Several students made a graph which showed 30 minutes time for each pound.

In problem 9 most of the students tried to get a straight line. Six students tried to construct a circle graph.

The units for the two axes proved to be the big obstacle in problem 10. Ten students used $3x+5$, $6x+10$, $9x+15$, for the vertical axis; and 1, 2, 3, 4, . . . to represent A on the horizontal axis.

TABLE XLX

Results of General Mathematics 2 on Test 11.

Problem	Correct	Incorrect	Omitted	Total
1.	23	2	2	27
2.	27	0	0	
3.	21	1	5	
4.	15	10	2	
5.	21	6	0	
6.	17	10	0	
7.	5	21	1	
8.	10	16	1	
9.	3	12	12	
10.	10	8	9	

TABLE IV

Summary of Survey Data for 1950

Year	1949	1950	1951	1952
1	10	10	10	10
2	10	10	10	10
3	10	10	10	10
4	10	10	10	10
5	10	10	10	10
6	10	10	10	10
7	10	10	10	10
8	10	10	10	10
9	10	10	10	10
10	10	10	10	10

Comments on Results of General Mathematics 2 Classes

on Test 11.- On problem 4 most of the incorrect answers were 76000. The students in this class did much better on problem 5 than those in General Mathematics 1. In problem 6 no one used a broken axis or broken diagram. The error most commonly made was that of giving equal intervals for 0 to 7000 and 7000 to 7010.

The most noticeable thing in the work on problem 8 was, that of those who obtained the correct graph, six had drawn an incorrect one at first and had then crossed it out. I think this shows that the pupils have learned something of value in General Mathematics 1 or in the early part of General Mathematics 2 - namely, to check their results. It is lamentable that more students in all classes do not learn to do this. The only check most students use is the answer in the back of the text.

TABLE XX

Results of General Mathematics 1 Classes on Test 111.

Problem	Correct	Incorrect	Omitted	Total
1.	8	76	1	85
2(a)	41	44	0	
(b)	32	50	3	
3(a)	78	5	2	
(b)	74	9	2	
4(a)	85	0	0	
(b)	83	2	0	
(c)	46	39	0	
5(a)	85	0	0	
(b)	71	2	12	
6.	52	31	2	

Comments on Results of General Mathematics 1 Classes

on Test 111.- As in other classes these students do not understand what has taken place between the second and fourth hours in problem 1.

In problem 2(a) eighteen students said the heat was turned on at 1 p.m., and twenty-one students said it was turned on at 2 p.m. In the first case they fail to observe that the graph descends for one minute and then for one minute there is no change; after this the graph begins to rise rapidly. The heat must have been turned on at 2 minutes after 1 p.m. or shortly before this. I accepted answers of 1.01 or 1.02 p.m.

Not many of the students could give an acceptable answer for 2(b).

In answering problem 3, two students said the graph should go up for awhile after the parachute opens.

A very interesting point is brought up by the answers to problem 4. Practically every student had the answer to 4(b) correct, yet thirty-two students said that the cost of food would be \$66. That is, they can correctly observe the relative differences in size when this problem is isolated, but when another factor is introduced, they fail to keep track of this relative difference. I have noticed that where numbers are introduced students have a tendency to do something with the numbers and then give a reason afterwards, if a reason is required.

In problem 6 most of the incorrect answers were, "It is not accurate enough." They are told that the graph accurately portrays the relative importance of the various industries; but, knowing they are expected to write something, and not knowing the correct reason, they give this answer as an easy way out.

TABLE XI

Results of General Mathematics 2 Classes on Test III.

Problem	Correct	Incorrect	Omitted	Total
1.	4	23	0	27
2(a)	24	3	0	
(b)	21	4	2	
3(a)	26	1	0	
(b)	26	1	0	
4(a)	27	0	0	
(b)	27	0	0	
(c)	22	4	1	
5(a)	27	0	0	
(b)	25	0	2	
6.	22	5	0	

Comments on Results of General Mathematics 2 Classes

on Test 111.- The students have learned to read graphs fairly well. The first problem however still causes difficulty. Some of the answers for problem one included these statements:

"The car stopped for one hour for repairs at 2 o'clock."

"The car slowed down between two and three o'clock."

"The car travelled 20 miles per hour for the first hour, 40 miles per hour for the second hour, 20 miles per hour for the third hour and then kept speeding up after that."

General Mathematics 2 students are much better than the first year class in problem 4(c). Several of them got the incorrect answer \$66 at first, but crossed it out and got the correct result. One student divided \$200 by 3 and then changed the decimal in order to get a reasonable answer, \$666.

General comments on the results of the tests in the two General Mathematics Classes.- General Mathematics 2 students show that they understand the terms: horizontal axis, round numbers, negative numbers, and ratio much better than the General Mathematics 1 class.

While the last three terms to be defined are not known any better by the second class still, the students realize they do not know. This indicates that they have learned something-- "'tis a wise fool who knows he is a fool." One would hardly expect the Mathematics 2 class to define these terms if they were not known before these pupils took Mathematics 1. In teaching General Mathematics 1, teachers

would avoid as many technical terms as possible.

It is surprising to find that so many of the General Mathematics 1 students do not really understand what the horizontal axis is. This deficiency is well taken care of later, for nearly all of the General Mathematics 2 students know the term.

The results of test 2 show that students in these classes cannot solve problems graphically. I believe that they should be able to solve these problems, and that it is our duty to find out how to give them this skill, so that they will succeed in work of this kind.

General Mathematics 1 students have much difficulty in reading graphs. The majority of General Mathematics 2 students can read graphs fairly well.

CHAPTER 1V

TEACHING GRAPHS IN GENERAL MATHEMATICS 1

After studying the results of the tests given to the various classes of students, I decided that the best approach to the topic of graphs was by the method of analysis. That is, I believe that by a study of many examples of various types of graphs, the student will learn the graphical language.

While it is true that one learns by doing, one must not forget that this is also the way one learns to do things incorrectly. Though this method fails to develop the creative instinct, still, some psychologists put little faith in transfer of training. Even if transfer of training is sure to take place, I believe the method is still the correct one to follow. Since we are dealing with pupils of lower than average intelligence, it is wise to teach them to understand what others create, rather than to try to develop a talent for original work.

Taking this method as a basis for my work in General Mathematics 1 classes, I started the course with newspaper and magazine pictograms. My reasons for using these are: they are interesting, they are the type of graph the students are most likely to meet outside the school room, they are easily obtained by the students, they well illustrate the good and bad features of graphs.

We secured picture graphs of all kinds -- armaments, wheat production, death rates, occupations, business increase,

and many others. The pupils did not need to be told what the good points of these graphs were. As an illustration of one of the bad points we took a graph which showed the relative standing armies of the world, previous to the war, depicted as soldiers in battle dress. Some of the pupils noticed the distortion of the true figures but others did not. We took the accompanying figures and showed that the comparisons were made for height alone. We then cut out two figures and traced them on squared paper. When the squares were counted, the pupils could see that where one was supposed to represent an army three times the size of another, its area was nine times the size. We took several other cases to show that the actual difference in size on the paper, is the square of the ratio of the heights.

We then took two similar shaped containers and filled the larger one with water. Then we filled the smaller one from the larger one. The pupils could see that the true difference where volumes were concerned is the cube of the ratio of the heights. (Incidentally, we had to take up the meaning of ratio.)

Then we went back to the picture of the soldiers. The pupils easily agreed that height alone was not an indication of the true strength of an individual but that the weight is more truly an indication. Thus the soldier who was three times as high as the other would tend to give the impression that he was twenty-seven times as strong as the other.

Then we showed how a bar graph could be used in place of the figure. To avoid the error in the picture graph we saw that the widths should all be the same. In this way height alone was the factor which we must use for comparisons.

The bar graph alone showed the comparative sizes but, unless a scale was introduced, we did not know the actual amounts. We then took up the placing of a scale on the vertical axis. In this regard, we took various examples of good scales and practised getting answers to different problems, based on the scale. Then we took up the case of several types of bad scales. Next we took different examples where the scales were omitted and the pupils filled in the scale. These scales were considered and good and bad points brought out.

Then we showed that, since height alone was the deciding factor in a bar graph, the tops of the bars were all that was necessary. We joined these with a broken line and so illustrated the broken line graph.

We took a number of broken line graphs and discussed what each one represented. Then the students drew broken line graphs based on information given them. The fact that each top of a bar gave us a point on the graph was brought out, to show that the line between the points had no meaning except as an aid to the eye in going from one point to the next.

At this point, the scale for the horizontal axis had

There are several ways in which the system of the future, if properly organized, may be made to work better than the present system. It is not possible to discuss all the ways in which the system may be improved, but the following are some of the most important ones. The first is to make the system more efficient. This may be done by making the system more flexible, so that it can adapt to changing conditions. It may also be done by making the system more economical, so that it can be operated at a lower cost. The second is to make the system more equitable. This may be done by making the system more democratic, so that the people have a greater say in the way it is operated. It may also be done by making the system more just, so that the benefits are distributed more fairly. The third is to make the system more effective. This may be done by making the system more powerful, so that it can do more things. It may also be done by making the system more reliable, so that it can be depended upon to do what it is supposed to do. The fourth is to make the system more attractive. This may be done by making the system more interesting, so that the people want to use it. It may also be done by making the system more useful, so that the people can get more out of it.

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to be considered. Examples of good scales were taken. Faulty scales were shown and the bad points discussed. Then problems were given on placing of scales on the axis to satisfy certain given conditions.

The same method was followed throughout the whole period in which graphs were taken up. Briefly it is: 1. Take up a great number of good examples. 2. Discuss the points carefully. 3. Test on reading this type of graph. 4. Point out errors that could easily be made. 5. Work out with the class certain problems involving the principle. 6. Set problems for the students. 7. Check the problems and discover errors. 8. Discover cause of errors and give method to avoid this error. 9. New set of problems with special emphasis on any weak points discovered in first test.

We then took up the following:

1. Cartograms

(The Alberta Wheat Pool Hail map is quite good).

2. Circle Graphs

3. Axes of Reference and Plotting Points

4. Straight Line Graphs

5. Solution of Problems by Straight Line Graphs

Then we took up some applications of graphical principles as:

The Stadia Hairs in a Transit

Angles of Elevation and Depression

Similar Triangle Method of Solution

to be considered, however, as the only one of its kind.

Other similar cases have been found in the same locality, but

none of them have been found in the same locality.

Other similar cases have been found in the same locality.

Other similar cases have been found in the same locality.

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Other similar cases have been found in the same locality.

Other similar cases have been found in the same locality.

Thermograph

Barograph

Evaluation of the material used in teaching graphs

in General Mathematics 1.- I believe that the most important part of the whole teaching program was that of concentrating on the study of actual graphs. Most of the graphs studied were taken from newspapers, magazines and books, which were brought to school by the students. Since they had brought these from their homes, they could realize the value of learning to read them. That is, they could see that they were learning something which they could use in actual life.

I believe that the study of these graphs is of high practical value. Students who have learned to read them will be able to glance at a graph in a newspaper and really understand what it represents. This ability is particularly valuable for the "C" class students, since they are noted for getting confused when they have to obtain their information from the printed page.

This is an age of speed and there is no doubt that graphs represent one of the best means of giving information quickly. If more people could read graphs intelligently, more graphs would be used.

Another value derived from the study of actual graphs is the training given to the pupil in detecting flagrant misrepresentation. Pictograms are the chief culprits in this regard. The correct interpretation is made when heights

alone are considered, but, to the unwary, the area is the basis on which conclusions are drawn. A few illustrations of this error will train the student to avoid this danger.

One error I made in teaching graphs in this class should be avoided. I used too many technical terms. The pupils learned these terms, but I believe the time could have been used to better advantage in other work. It is much better to be able to do something correctly than to be able to use correctly the technical terms which may be applied to the problem.

Some of the examples of graphs were too difficult for the students. This was unwise as it tended to confuse some of them. This class generally need to develop more self confidence, so problems should not be given which are too difficult for them.

The example of the use of graphical methods to determine distance, as shown by the stadia hairs in a level, proved to be quite useful. Of course the level itself aroused interest and the students were anxious to use it. After they had learned to estimate distances with it, they were given an explanation of the principle involved. Then they used a ruler held at arm's length to further illustrate this device. They easily learned how to estimate various heights and distances.

I must admit that this method required quite a lot of time. More time was spent on this topic than would be

allowed in other classes. I believe that the extra time spent is justifiable since we are dealing with pupils of lower than average ability in school work. We must choose between two evils. We must take more time than is given in other classes, or we must be satisfied with inferior results. I prefer the former. "C" grade students generally lack self-confidence and sure knowledge of a subject is one of the best ways to restore confidence.

The pupils learned to read graphs and to use simple applications of them in their everyday life. This, I believe, was the chief value obtained from the course.

CHAPTER V

FINAL TESTS TO ALL CLASSES

After the topic "graphs" had been taken, in the manner just described in Chapter IV, I gave final tests to the class. First I gave the three tests which had been given earlier in the year. Then I gave a new set of tests to this class.

I gave tests 1, 2, and 3 to various classes that had finished the topic "graphs" for the year in their regular school course.

I gave both sets of tests to Algebra 1 classes after they had completed work on the topic "graphs" for the year.

The reasons for giving all these final tests were:

1. I wanted to secure a comparison of the two sets of tests.
2. I wished to judge the progress made by various classes in a year's work as shown by their results on the two sets of tests.
3. I wanted a comparison between General Mathematics students who had not followed my method and my own students.
4. I expressly wished to see if General Mathematics students were capable of forming the graphical concept.

The results of the two sets of tests, which were given to Algebra 1 classes, show that the second set of tests is more difficult than the first. This is as I wished, for I wanted to avoid the criticism that the second set of tests was easier than the first.

I did not have many students in General Mathematics so I included in this testing program all students who had obtained a "C" grading. The "C" grade students, who were not regular General Mathematics students, would not have better mathematical ability than those who were regular students.

I am giving all the results of these final tests in graphical form. I list below the titles of the various graphs.

1. Comparison of the Results of Preliminary Test I with Final Tests I and IV in the General Mathematics I Class in My Own School
2. Comparison of the Results of Preliminary Test II with Final Tests II and V in the General Mathematics I Class in My Own School
3. Comparison of the Results of Preliminary Test III with Final Tests III and VI in the General Mathematics I Class in My Own School
4. Comparison of the Results on Test I of Similar General Mathematics I Classes Before and After the Topic "Graphs" Had Been Taken
5. Comparison of the Results on Test II of Similar General Mathematics I Classes Before and After the Topic "Graphs" Had Been Taken
6. Comparison of the Results on Test III of Similar General Mathematics I Classes Before and After the Topic "Graphs" Had Been Taken
7. Comparison of the Results on Test I of Similar

Algebra 1 Classes Before and After the Topic "Graphs" Had Been Taken

8. Comparison of the Results on Test 11 of Similar Algebra 1 Classes Before and After the Topic "Graphs" Had Been Taken

9. Comparison of the Results on Test 111 of Similar Algebra 1 Classes Before and After the Topic "Graphs" Had Been Taken

10. Comparison of the Results on Test 1 of Similar Grade 1X Classes Before and After the Topic "Graphs" Had Been Taken

11. Comparison of the Results on Test 11 of Similar Grade 1X Classes Before and After the Topic "Graphs" Had Been Taken

12. Comparison of the Results on Test 111 of Similar Grade 1X Classes Before and After the Topic "Graphs" Had Been Taken

13. Comparison of the Results on Test 1 of Similar General Mathematics 2 Classes Before and After the Topic "Graphs" Had Been Taken

14. Comparison of the Results on Test 11 of Similar General Mathematics 2 Classes Before and After the Topic "Graphs" Had Been Taken

15. Comparison of the Results on Test 111 of Similar General Mathematics 2 Classes Before and After the Topic "Graphs" Had Been Taken

16. Comparison of the Results of Test I and IV in Algebra I Classes After the Topic "Graphs" Had Been Taken

17. Comparison of the Results of Test II and V in Algebra I Classes After the Topic "Graphs" Had Been Taken

18. Comparison of the Results of Test III and VI in Algebra I Classes After the Topic "Graphs" Had Been Taken

Test 1V - Definitions

Give a definition of each of the following terms.

Give an example if it will aid you in defining the term.

- (a) Circle graph
- (b) Cartogram
- (c) Origin
- (d) Vertical Axis
- (e) Slope
- (f) Broken line Graph
- (g) Range (As applied to a graph)
- (h) Unequal Units
- (i) Proportion
- (j) Direct Variation
- (k) Best Fitting line
- (l) A Constant

Test V - Graphical Problems

1. In a properly drawn bar graph, if a bar 1 inch high represents \$2000, what will a bar $2\frac{1}{2}$ inches high represent?

2. A man earns \$4 per day. You wish to make a line graph to show the amount he earns in any number of days up to 10. Sketch a graph, showing the scales used.

3. If the taxes of a city are spent by seven departments, and you wish to make a graph which shows clearly the percentage of the total spent by each department, what kind of a graph would you use?

(a) Circle graph

(b) Broken line graph

(c) Straight line graph.

4. $\frac{698.47 \times 7.133}{497.86}$ approximately equals _____ .

Show your work.

5. If your paper is ruled in $\frac{1}{8}$ inch squares and the numbers in your table range from 0 to 50, you should let 1 inch represent what amount?

6. If the temperature recording for a two-week period ranges from -40 to 5, where should the horizontal axis be placed? Use a sketch to illustrate.

7. A car leaves Calgary at 9 a.m. for Edmonton, a distance of 200 miles, and travels at a speed of 40 miles per hour. At 11 a.m., an aeroplane leaves Edmonton for Calgary and travels 100 miles per hour. Show by a graph the positions

of the two, and when and where the plane will fly past the car.

8. If a river flows 2 miles per hour and a man rows 4 miles per hour in still water, show by means of a graph his positions at all times as he rows ten miles up the river and then rows back to his starting point.

9. A Power Co. makes a charge of \$1.50 per month for service. It charges 10¢ per k.w. for the first 10 k.w. Then it charges 5¢ per k.w. for the next 20 k.w., and 2¢ per k.w. for any amount over this. Show by a graph the charges which will be made for any amount of electricity used from 0 to 100 k.w.

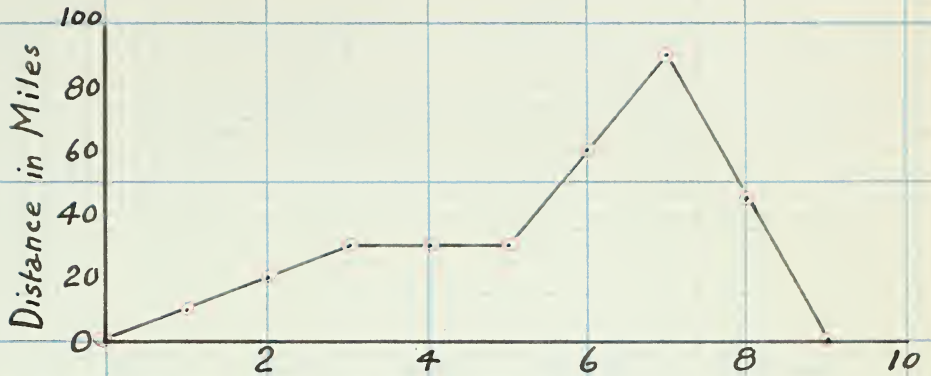
10. $A = \frac{1}{2}s - 4$

Draw the graph of this equation for all values of s from 0 to 6.

at the top, and the bottom of the page is the same. The text is written in a very small, dense, and somewhat illegible script. The page appears to be a continuation of a document, with the page number '-2-' at the top center. The text is arranged in several paragraphs, with some lines indented. The overall appearance is that of a scanned document with low contrast and some noise.

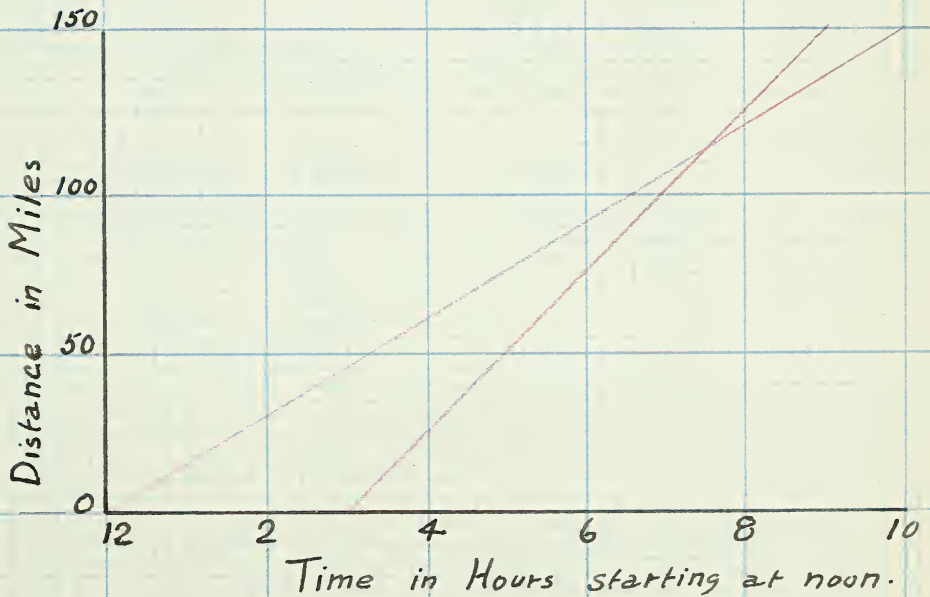
Test 6 Interpretation of Graphs

1.



Describe fully what the above graph tells you.

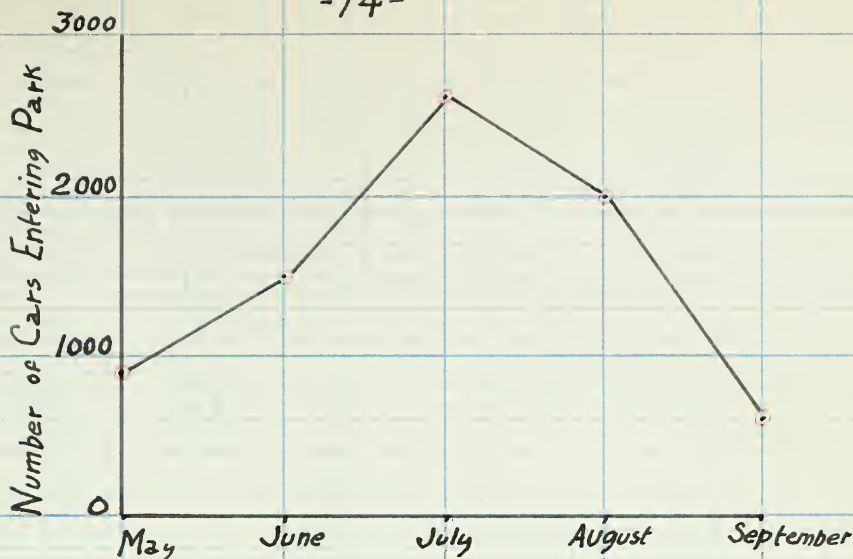
2.



(a) How far apart are the cars at 4 o'clock?

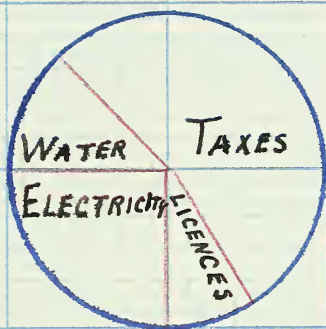
(b) When were they 15 miles apart?

3.



- How many cars entered the park on July 15th?
- How many more came in June than in September?

4.

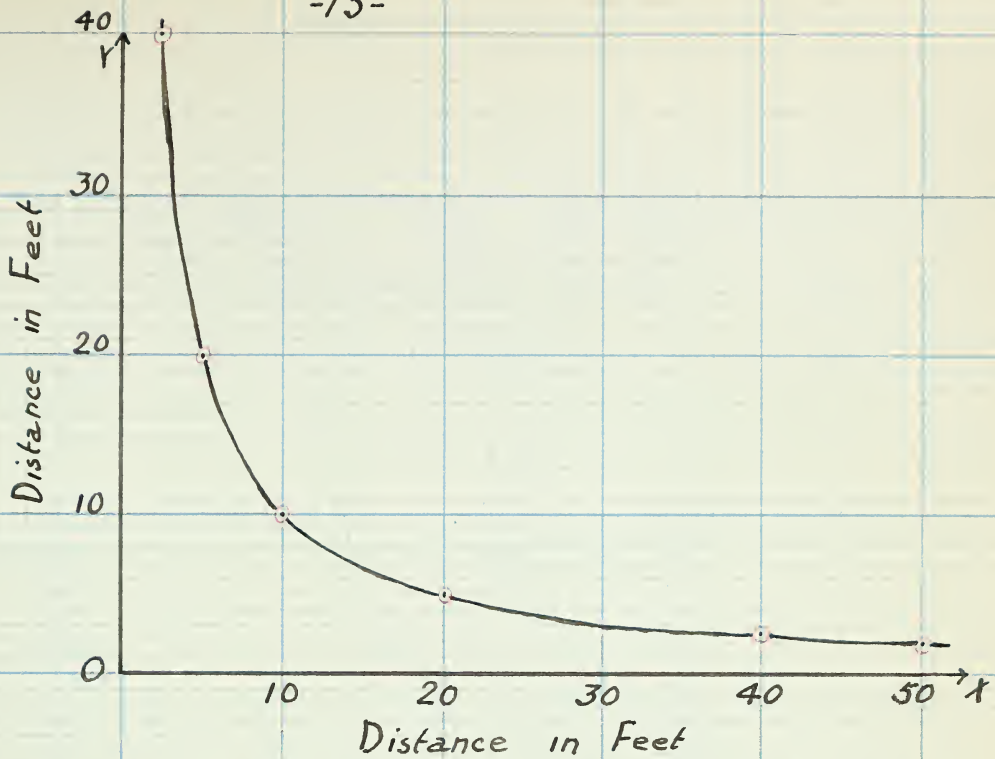


This graph represents the total income of a town for a year.

- If \$1000 is collected for water how much is the total income?
- How much more is paid for electricity than for water?
- How much is paid in taxes?

-75-

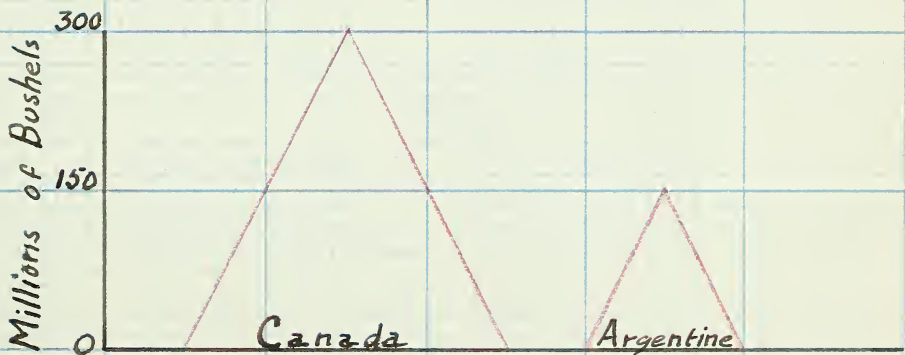
5.



(a) Give the equation showing the relation between X and Y .

(b) At what value does $X = Y$?

6.



This graph accurately portrays the amount of wheat exported one year.

Why is it not a good graph?

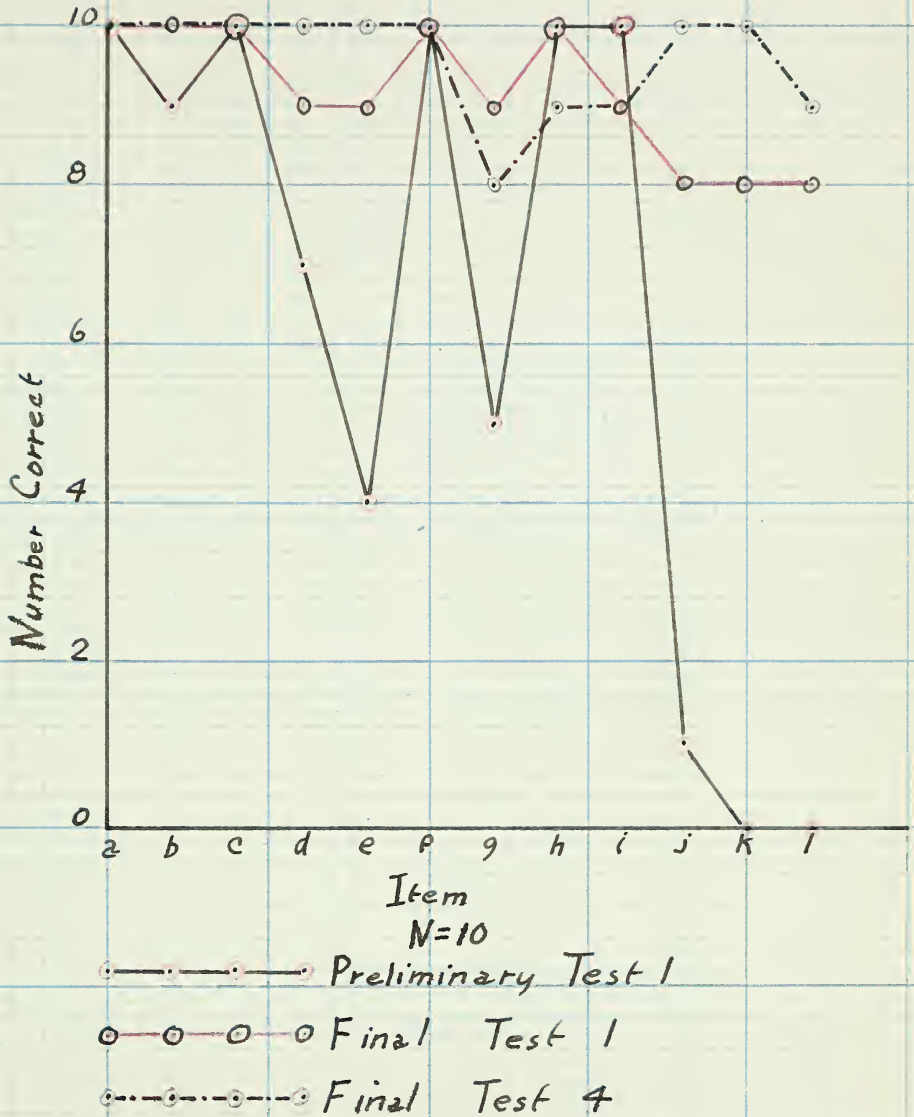


Fig. 1.— Comparison of the Results of Preliminary Test I with Final Tests I and IV in the General Mathematics Class in My Own School.

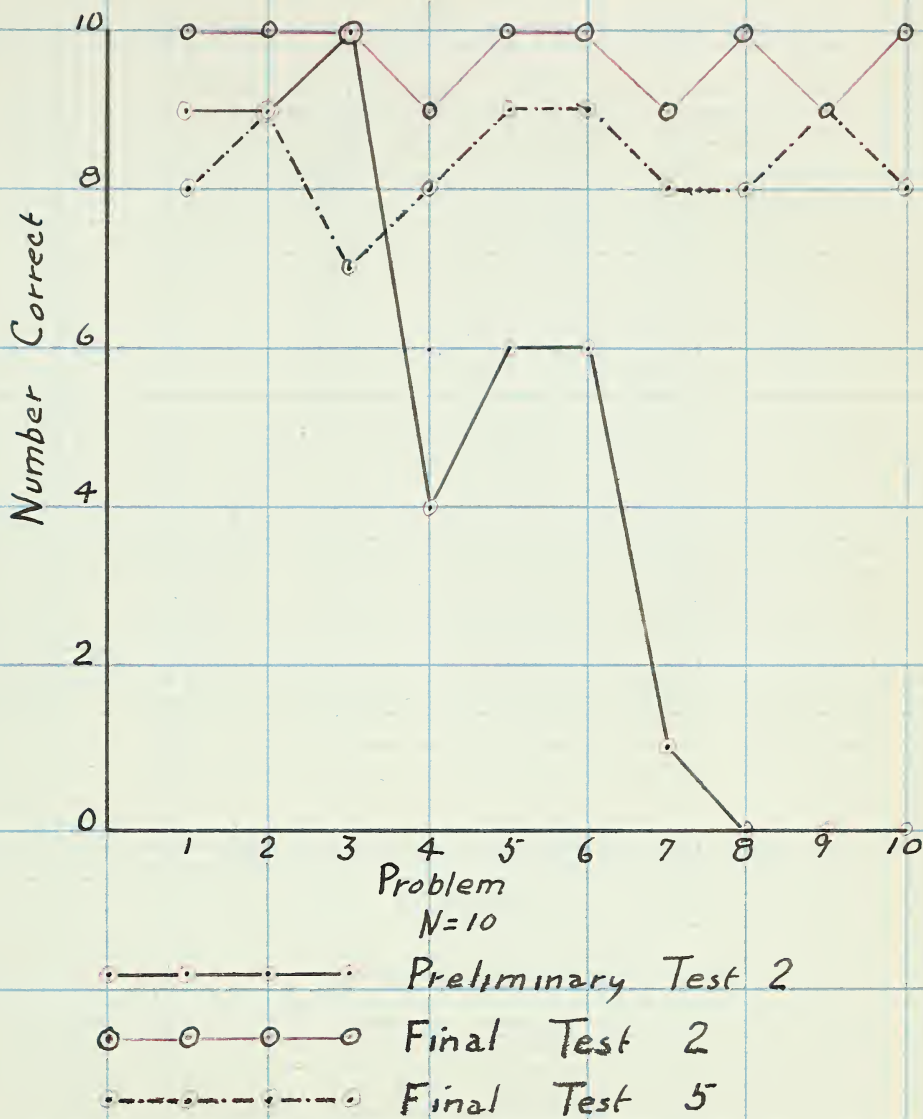


Fig. 2.- Comparison of the Results of Preliminary Test 2 with Final Tests 3 and 5 in the General Mathematics Class in My Own School.

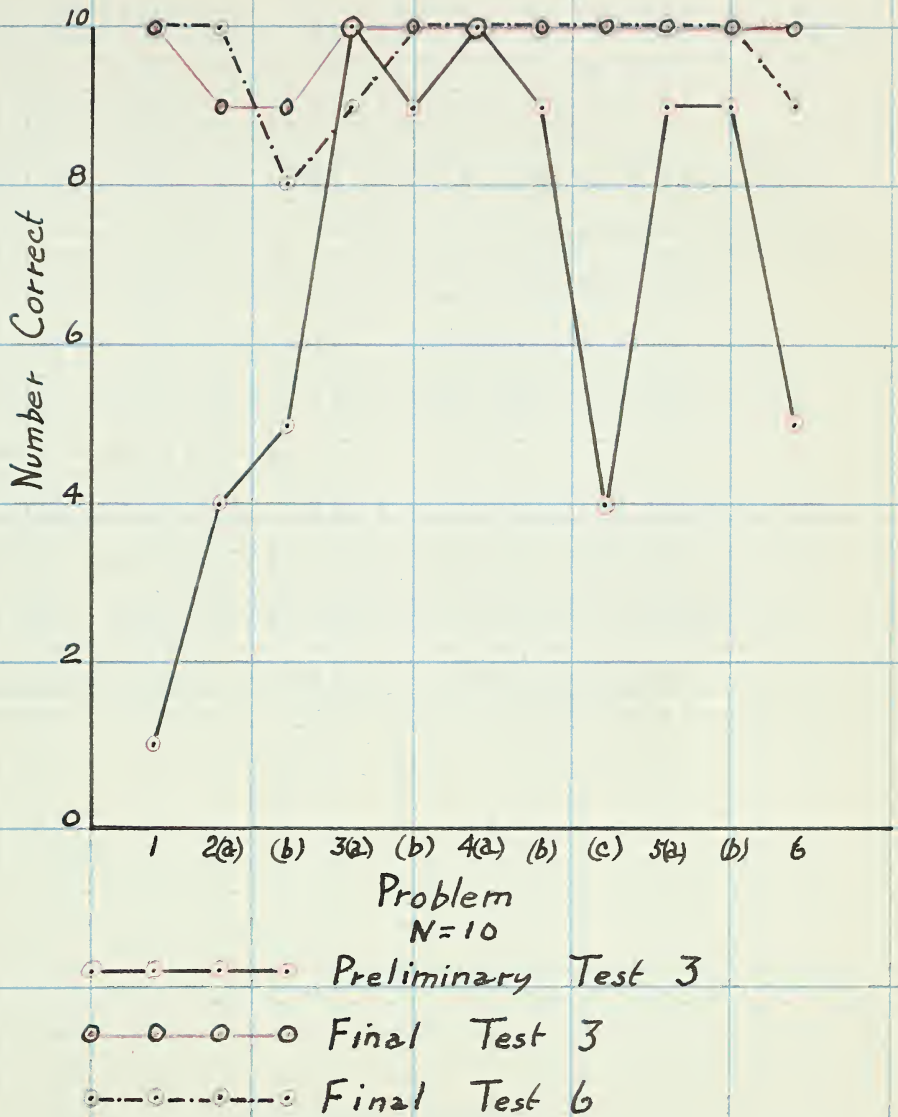


Fig. 3.— Comparison of Results of Preliminary Test 3 with Final Tests 3 and 6 in the General Mathematics Class in My Own School.

Comments on the results of final Tests I and IV in General Mathematics I.- The students have learned the meaning of the terms fairly well. They took test one early in the year and then tried it again five months later.

In test two the pupils show a very marked improvement. Of course, they may remember the solutions given at the time of the first test; in fact, the lower marks on test four would indicate this. Nevertheless, the results on the final tests two and four, show that the pupils can draw graphs reasonably well.

Results of tests three and six give us evidence that the students have learned to read graphs. Problem 5(a) in test six still gives difficulty but this is a problem in equations rather than a problem in reading graphs.

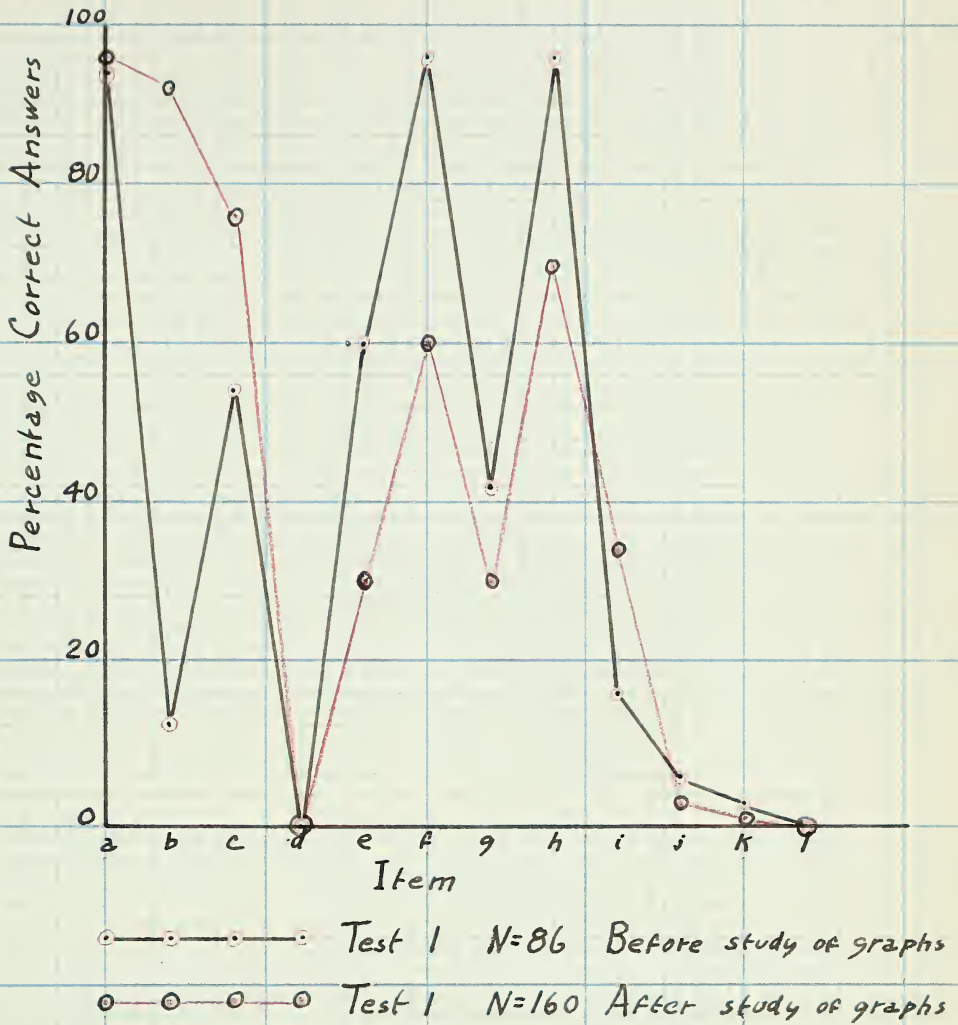


Fig. 4.- Comparison of Results on Test 1 of Similar General Mathematics I Classes Before and After the Topic "Graphs" Had Been Taken.

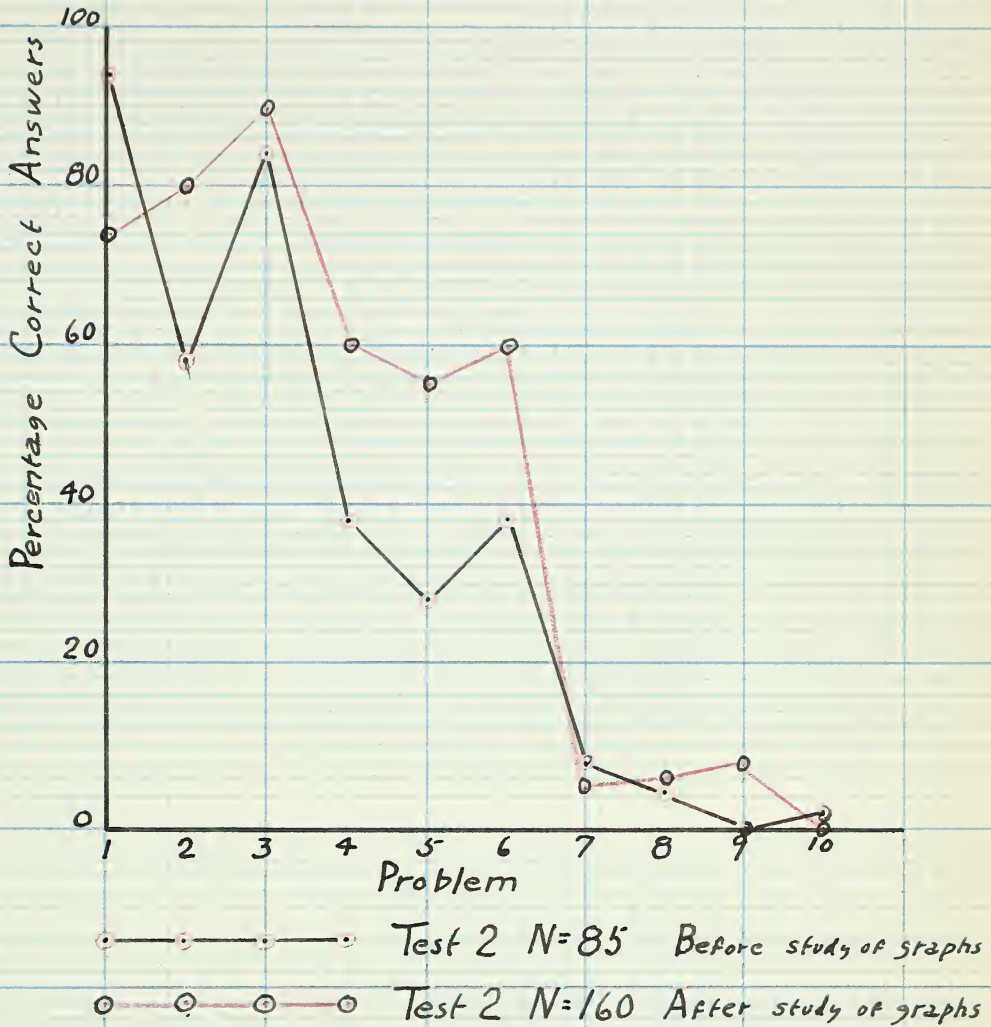


Fig. 5.- Comparison of Results on Test 2 of Similar General Mathematics 1 Classes Before and After the Topic "Graphs" Had Been Taken.

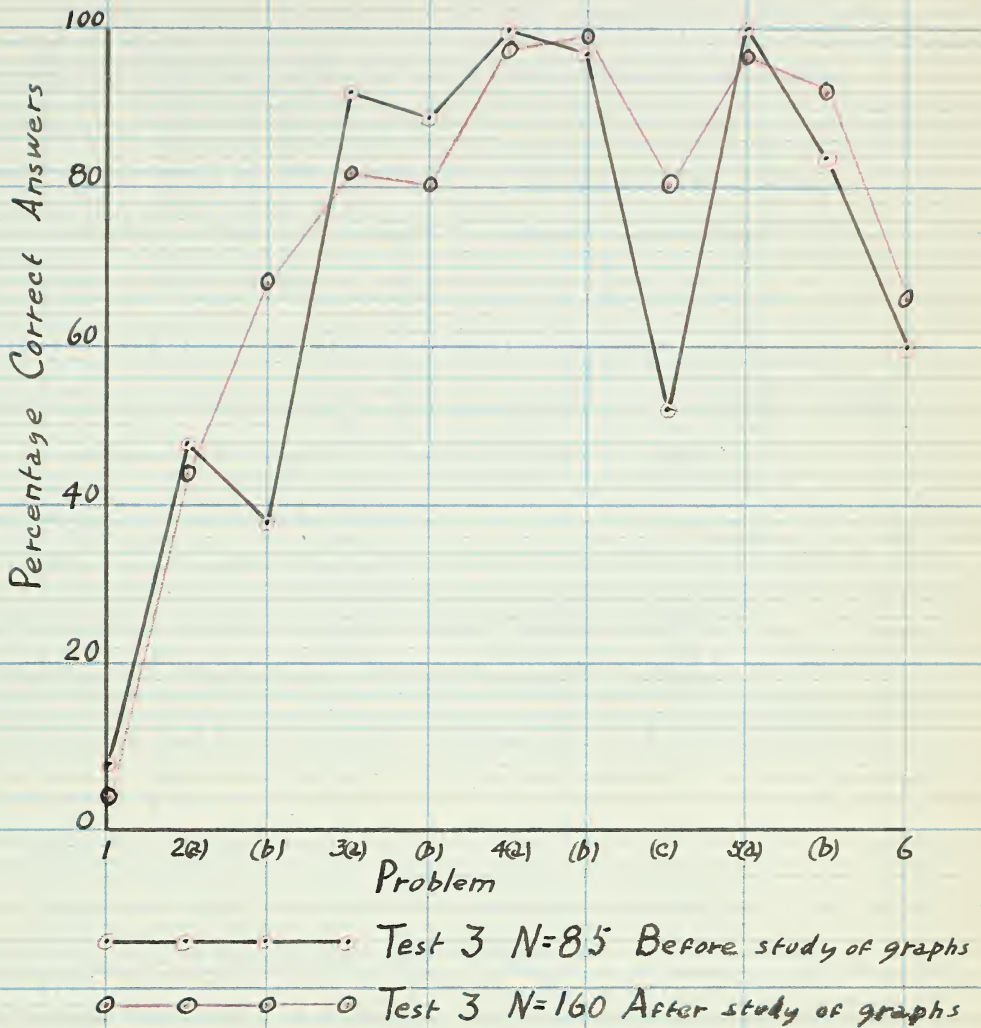


Fig. 6.- Comparison of Results on Test 3 in Similar General Mathematics I Classes Before and After the Topic "Graphs" Had Been Taken.

Comments on the results of similar classes in General Mathematics 1 on Tests 1, 11 and 111, given early in the year to the first class, and after graphs were studied in the other.- Some of these students were "C" grade students, while some were "A" and "B" students who were taking the commercial course. The "A" and "B" students had about fifty per cent more correct answers than the "C" students.

Two definitions which proved to be quite difficult were those for a "round number" and a "negative number". Some answers given as definitions of a round number were:

"Zero is a round number."

"360 is a round number."

"A number divisible by 2 is a round number."

"A number divisible by $2\frac{1}{2}$ is a round number."

Even more varied were the answers given for the definition of a negative number:

"Negative numbers are 1, 3, 5, 7."

"Numbers divisible by 2 are negative numbers."

"It means subtraction; as, $451-28$."

"It is the right number."

"It is not the right number."

"A negative number has a + ."

"A negative number is 3- ."

"It cannot be divided if it is negative."

"It is the opposite number."

"It is a number more than one."

"It is all numbers after the decimal."

"It is a number only divisible by itself and 1."

"It is a number that isn't there."

It is plainly evident that these General Mathematics students do not understand the meaning of negative. This is shown in problem 1, Test 111, as well, for very few students realized that the direction of the vehicle had changed.

Poor spelling was found in all grades, but it was particularly noticeable in this class. Here are a few examples of the ways in which "graph" was written: graff, grafp, grahp, graffe, graf, graft.

In problems 7 and 8 in Test 11, several students correctly plotted the points and then tried to make a straight line fit these points and zero. Several students said that if the points had been plotted accurately the graph would have been a straight line.

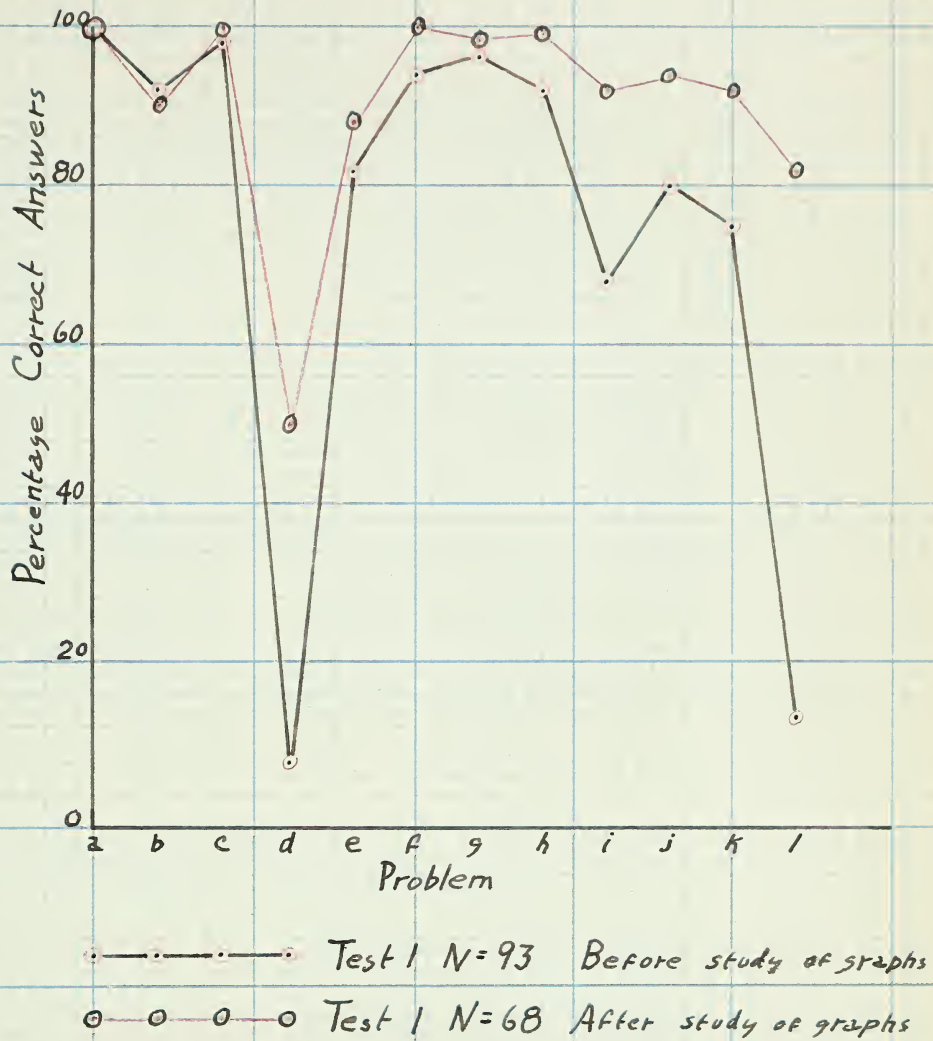


Fig. 7.- Comparison of the Results on Test 1 of Similar Algebra I Classes Before and After the Topic "Graphs" Had Been Taken.

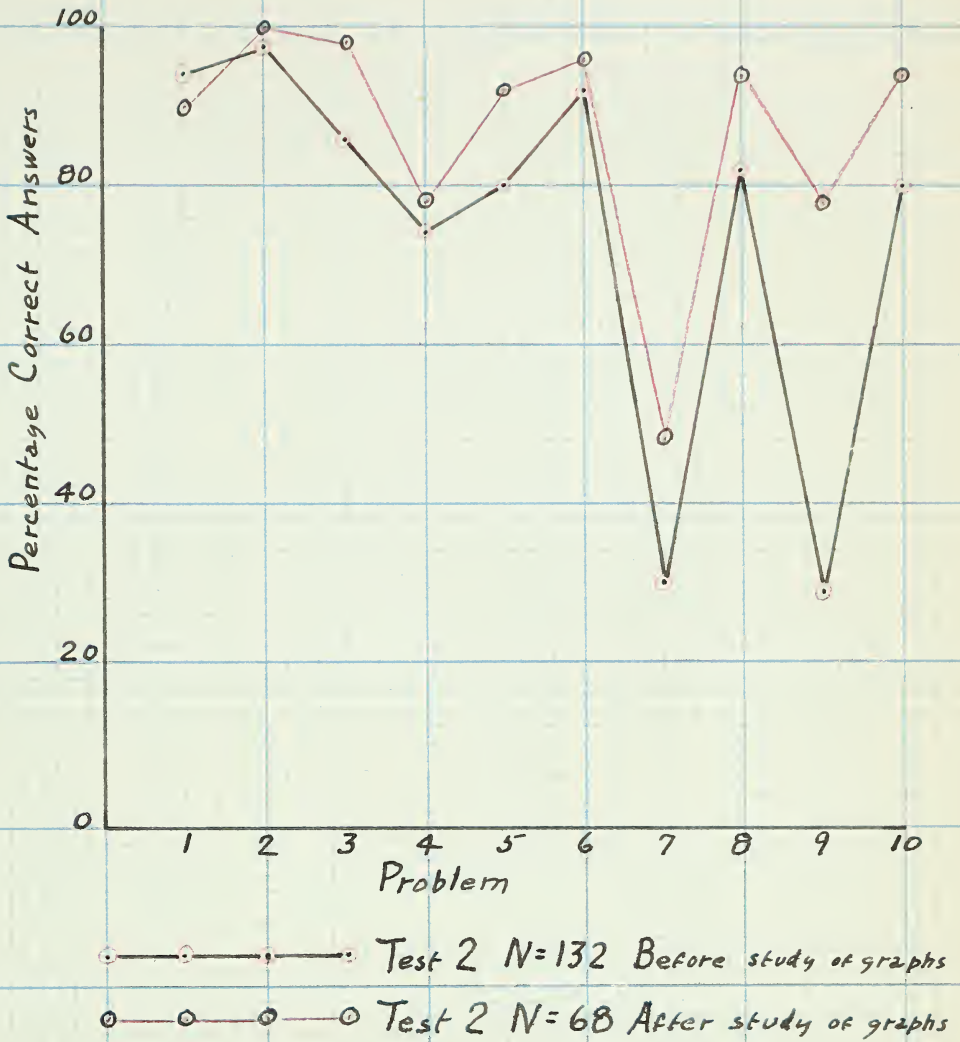
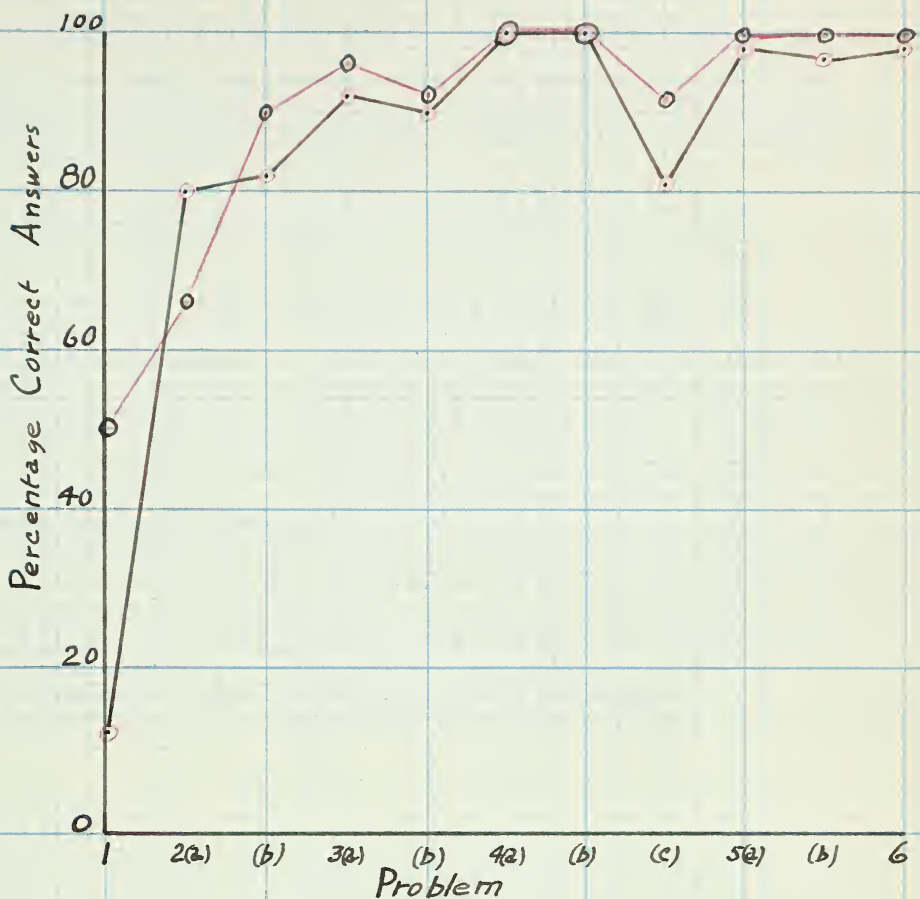


Fig. 8.— Comparison of the Results on Test 2 of Similar Algebra 1 Classes Before and After the Topic "Graphs" Had Been Taken.



—○— Test 3 N=131 Before study of graphs
 —○— Test 3 N=68 After study of graphs

Fig. 9.- Comparison of the Results on Test 3 of Similar Algebra I Classes Before and After the Topic "Graphs" Had Been Taken.

Comments on the results of the comparison of
Algebra 1 classes before and after studying graphs.-
Apparently the use of a broken axis (or broken diagram) is not stressed very much in schools. Undoubtedly this device has its dangers but it has its value as well.

A considerable number of students still confuse ratio with radius. The students have a great deal of difficulty with problem 7, test 11. They generally make the mistake of starting both cars at the same place. In problem 9, test 11, most of the errors are due to the pupils' efforts to make a straight line graph.

Problem 1, test 111, still proves to be too difficult for the majority of students in this class. The idea of the negative direction does not seem to be understood.

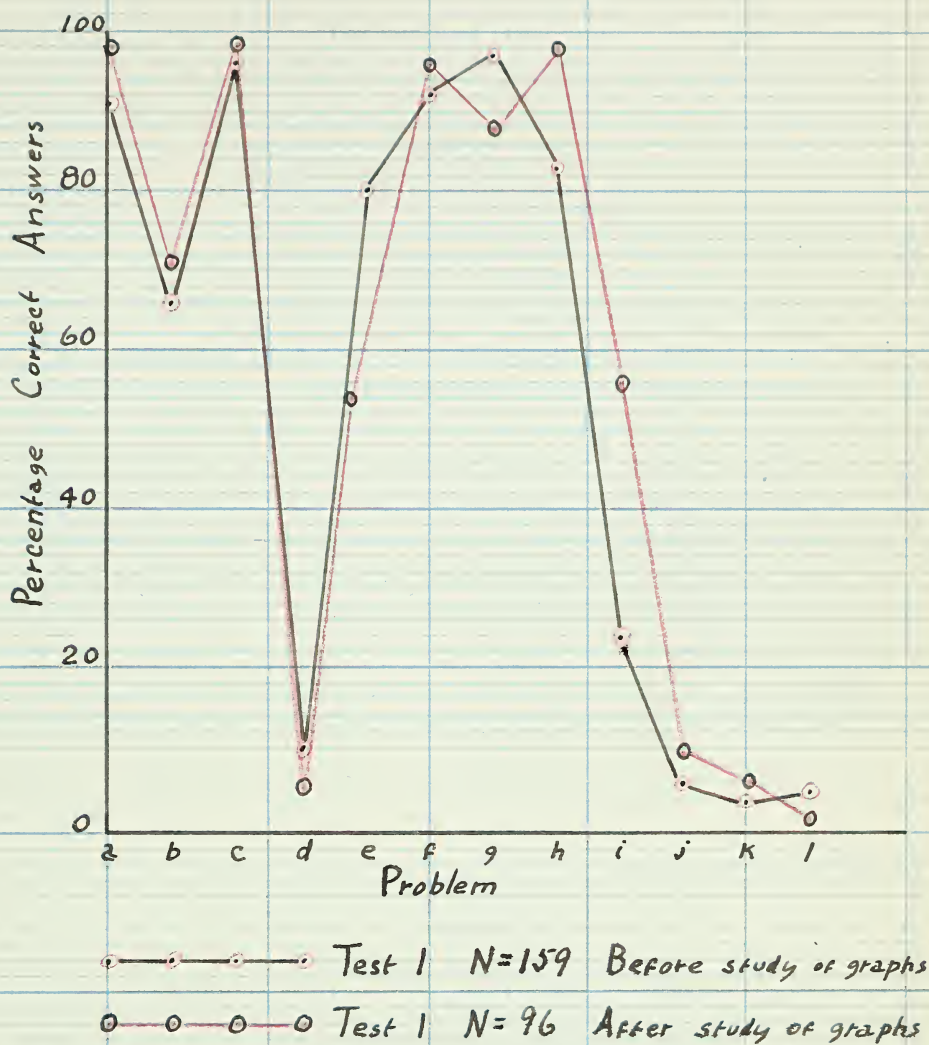


Fig. 10.— Comparison of the Results on Test I of Similar Grade IX Classes Before and After the Topic "Graphs" Had Been Taken.

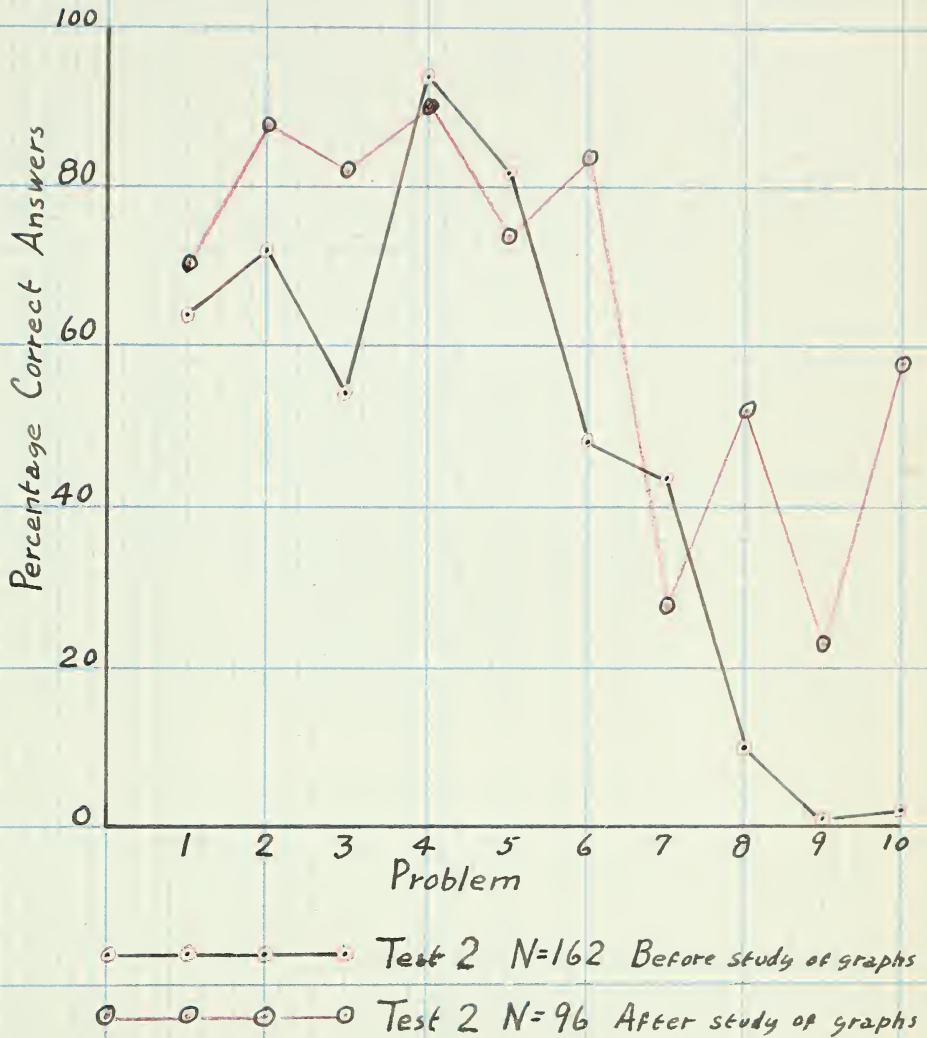


Fig. 11.- Comparison of the Results on Test 2 of Similar Grade IX Classes Before and After the Topic "Graphs" Had Been Taken.

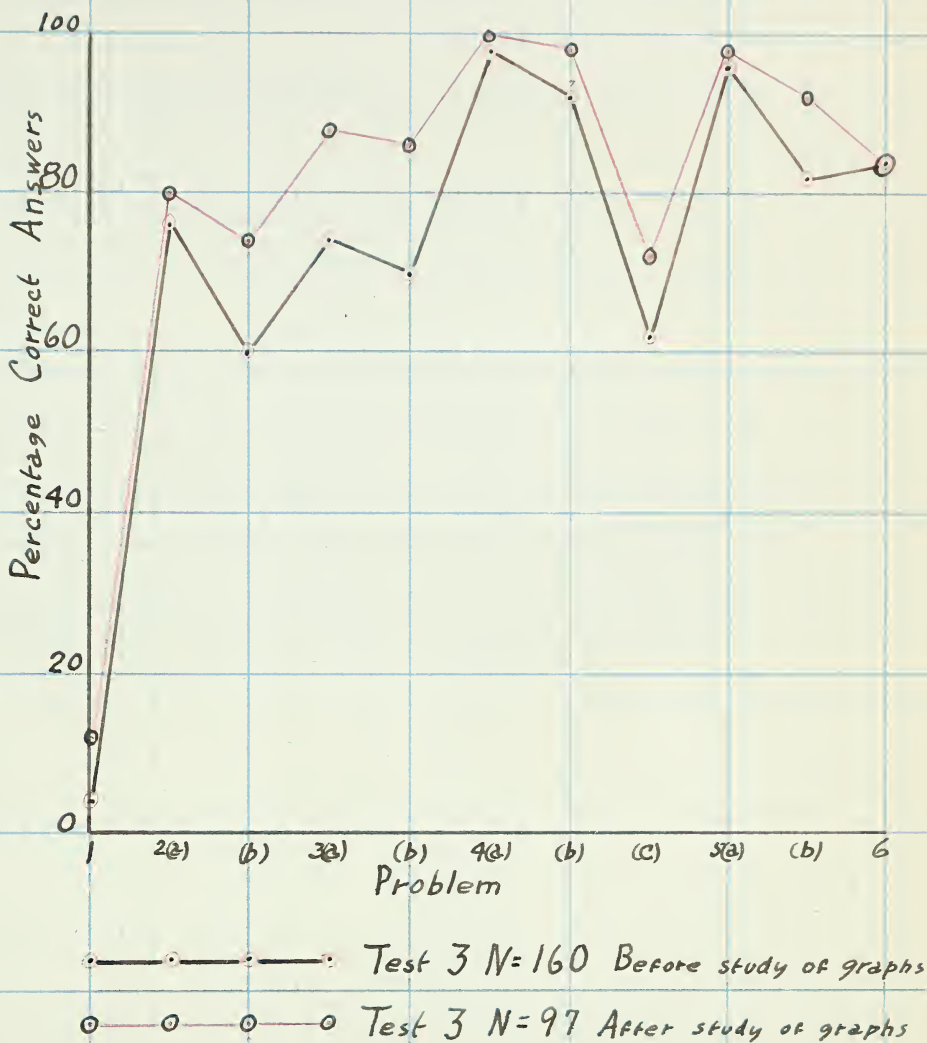


Fig.12.- Comparison of the Results on Test 3 of Similar Grade IX Classes Before and After the Topic "Graphs" Had Been Taken.

Comments on the results of the comparison of Grade 1X classes before and after studying graphs.- On the test in definitions there is little improvement in the results of classes that have taken the topic of graphs. The results of test 11 show that there is a fairly good improvement in ability to solve problems graphically. The pupils show a better reading knowledge of graphs after having taken the topic in this grade.

I believe that the results of these tests show that more attention should be given to the topic of graphs in this grade. Many topics covered in a short while may make a subject more interesting, and for the clever student may lead to wise stimulation of mental activity; but for the average student this method is not conducive to clear thinking.

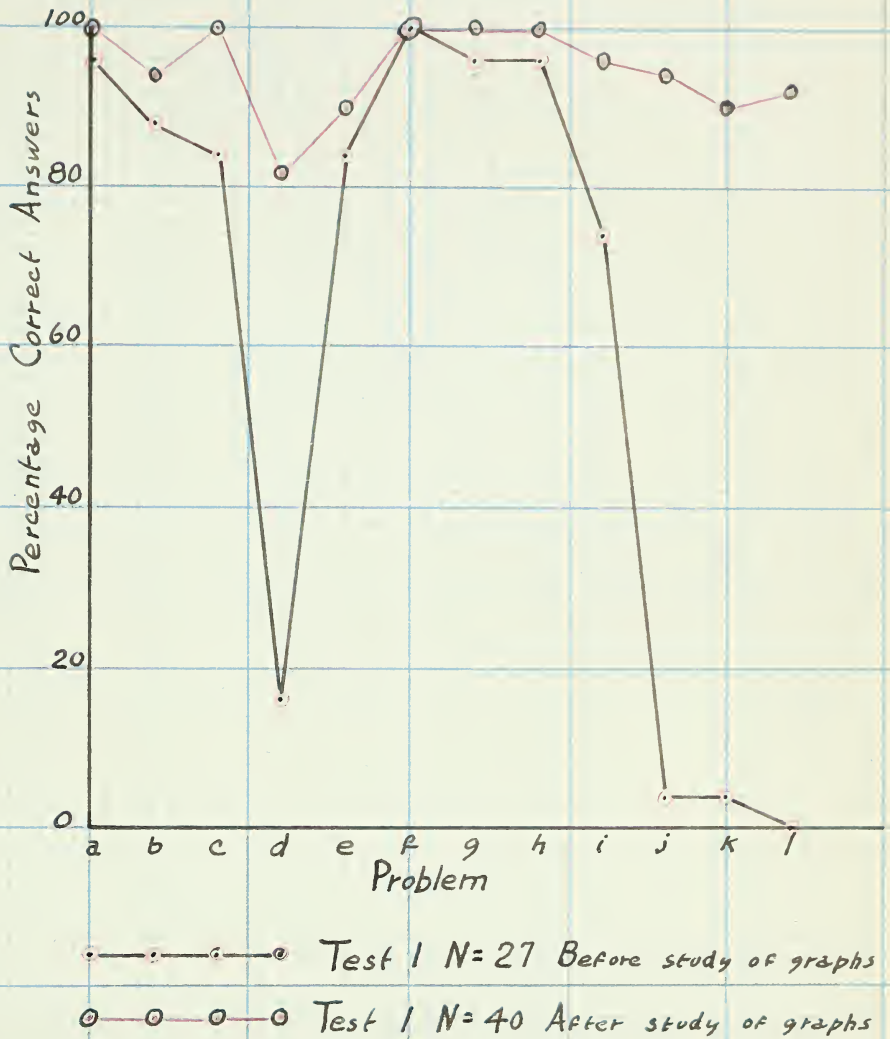


Fig. 13.— Comparison of the Results on Test 1 of Similar General Mathematics 2 Classes Before and After the Topic "Graphs" Had Been Taken.

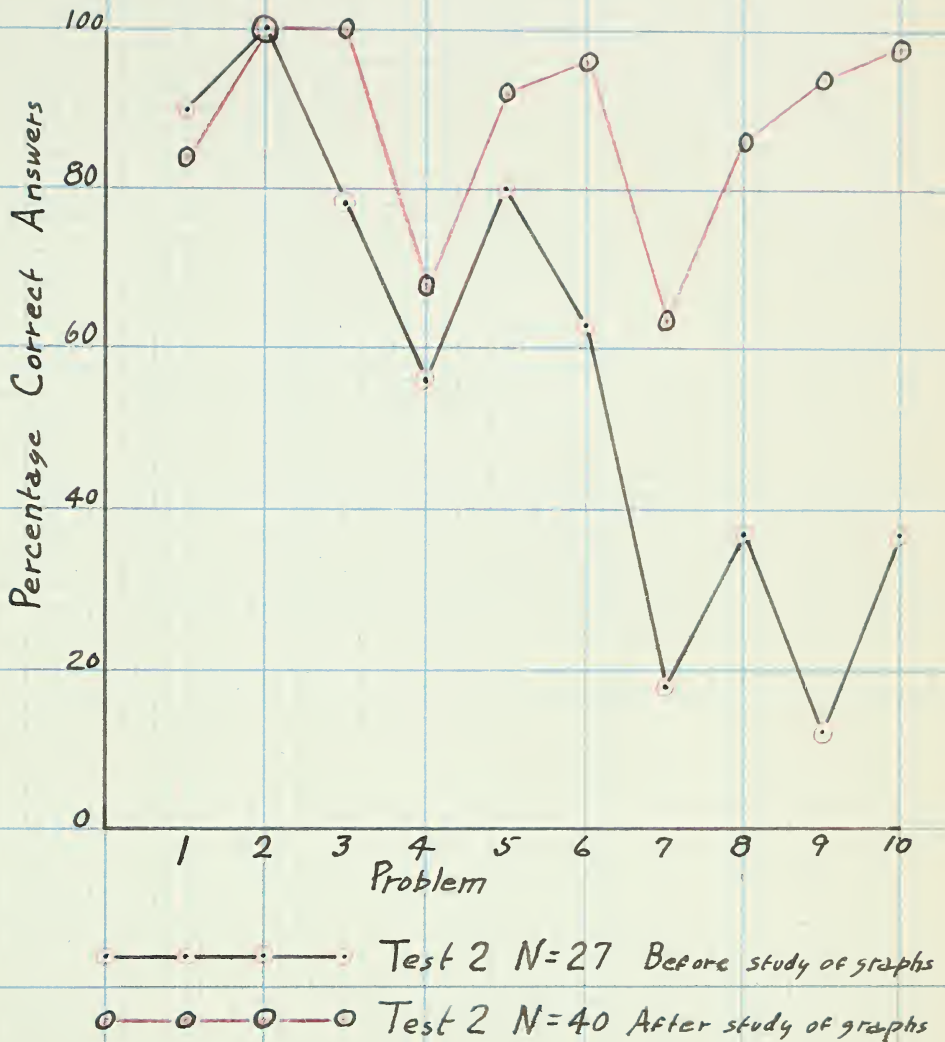


Fig. 14.- Comparison of the Results on Test 2 of Similar General Mathematics 2 Classes Before and After the Topic "Graphs" Had Been Taken.

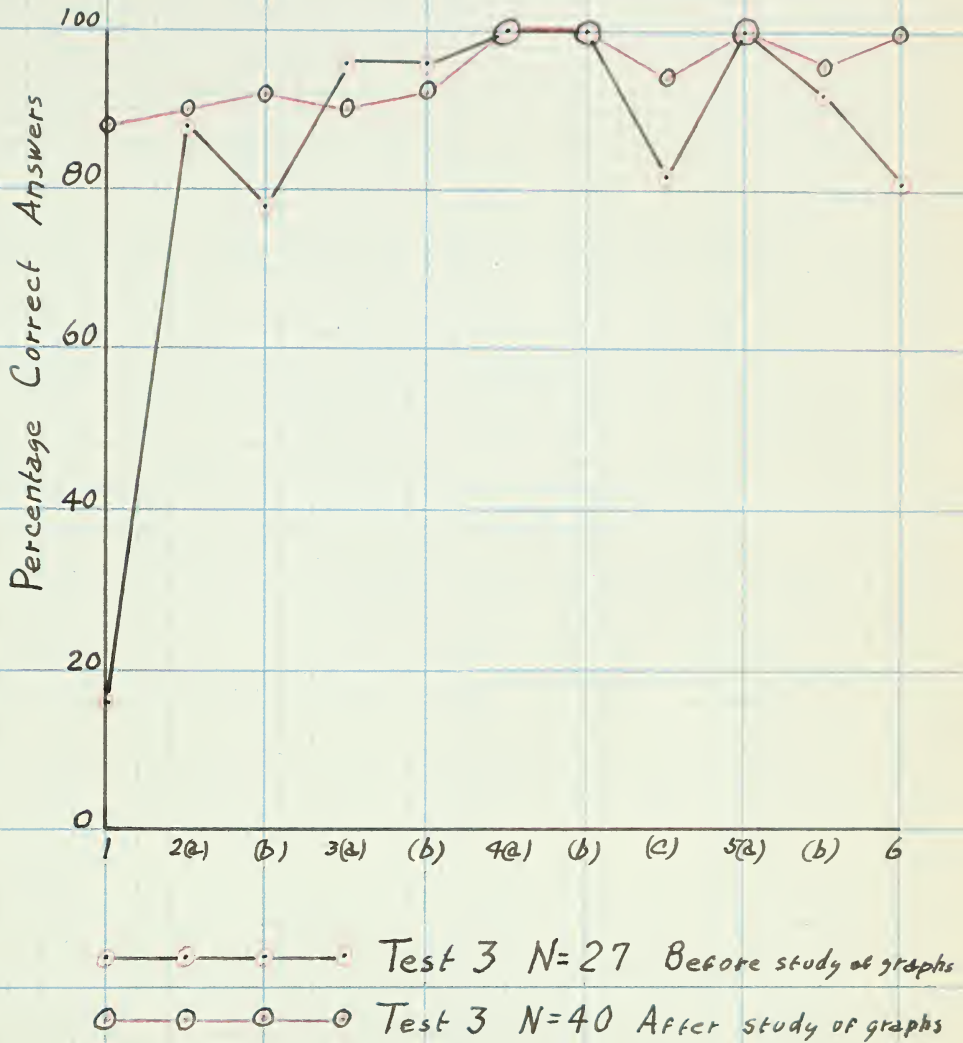


Fig. 15.- Comparison of the Results on Test 3 of Similar General Mathematics 2 Classes Before and After the Topic "Graphs" Had Been Taken.

Comments on the results of the comparison of General Mathematics 2 classes before and after studying graphs.- In General Mathematics 2 the topic "graphs" is given an important place in the year's work, and the results of the tests show that there is a corresponding increase in the standard of work done.

The classes were rather small in comparison with some of the others but I believe they are large enough to give us fairly reliable information. The results on all three tests are of a reasonably high standard. The extra time spent on this topic, in this class, proves that it pays to devote enough time to a topic to make it clear.

I believe that most mathematics teachers will agree that the graph is one of the most useful devices that can be employed to clarify a problem. When one draws a graph, he not only puts his problem in a form which can be understood by others, but he probably clears up his own doubts at the same time.

In this more or less orphan subject on our curriculum we find very well handled a topic which is very poorly treated in the other mathematics courses of the high school.

The students in General Mathematics 2 are not likely to have a higher-than-average I.Q. If the problem of graphs can be mastered so well in this class, it should not prove too difficult for other classes.

Graphs are well handled in this class. The

unfortunate thing is that so few students take General Mathematics 2. This optional course is given in only a few centres, for the majority of schools seem to regard the General Mathematics courses as not being of value sufficient to warrant their inclusion in a crowded curriculum. I think otherwise; I believe that everyone should learn some mathematics in high school even if the course is not a standard mathematics course.

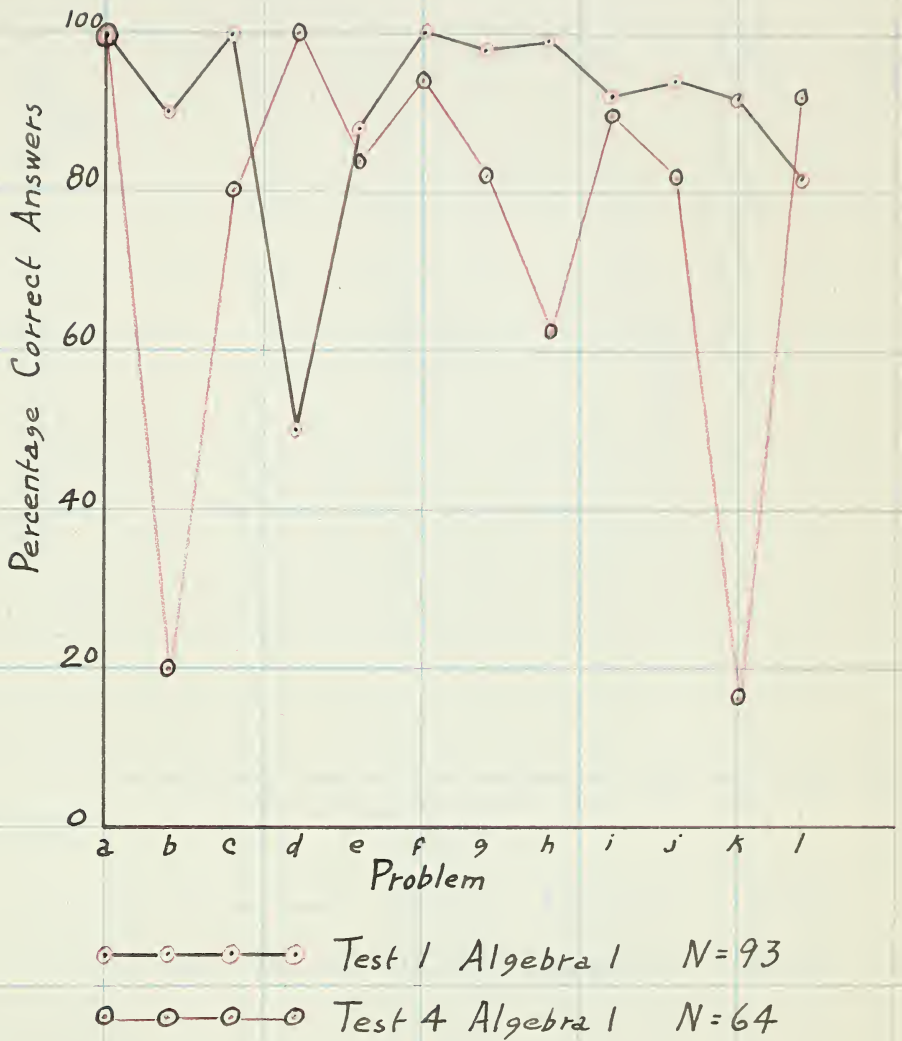


Fig. 16.- Comparison of the Results of Test 1 and 4 in Algebra 1 Classes After the Topic "Graphs" Had Been Taken.

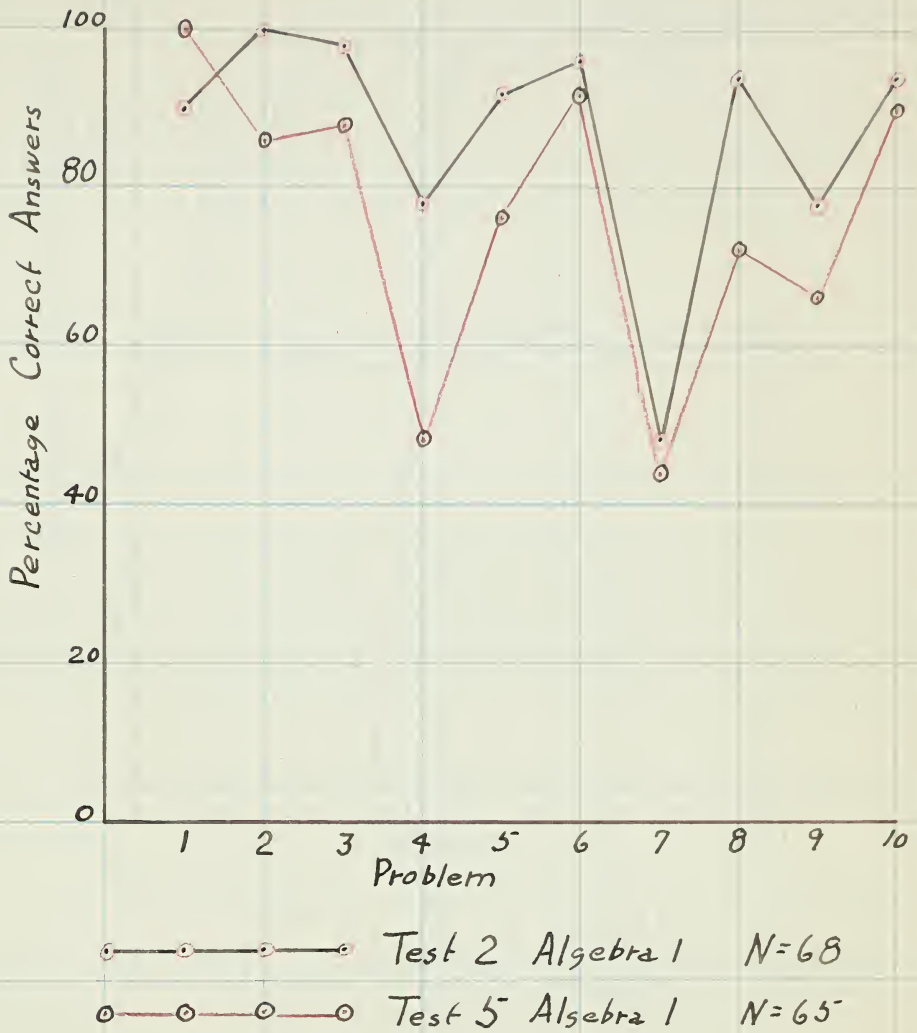
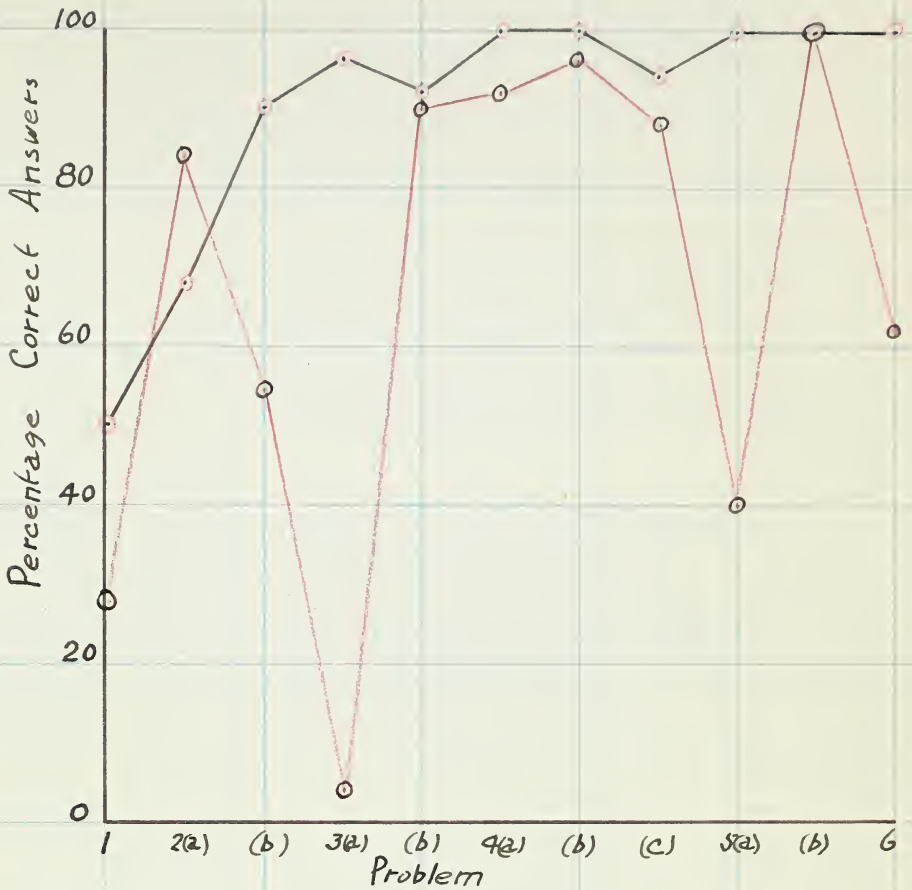


Fig. 17.- Comparison of the Results of Test 2 and 5 in Algebra 1 Classes After the Topic "Graphs" Had Been Taken.



●—●—●—● Test 3 Algebra 1 N=68

○—○—○—○ Test 6 Algebra 1 N=65

Fig. 18.- Comparison of the Results of Test 3 and 6 in Algebra 1 Classes After the Topic "Graphs" Had Been Taken

Comments on the comparison of the results of the first set of tests with those of the last set as given to Algebra 1 classes.- The second set of tests proved to be more difficult than the first set. This indicates that graphical terms and problems are not well understood by this class. It is not a case of poor results due to an unrepresentative type of test but of poor results due to a distinct lack of knowledge.

The straight line graph still proves to be a difficult one to read. The students fail to grasp the idea of the negative direction.

The students can draw a graph of a given equation reasonably well, but the fact that they cannot read a similar graph, proves that the problem has been solved by routine methods, not by intelligent reasoning.

CHAPTER VI

CONCLUSIONS

The results of the various tests I have made show that the problem of graphs has not been very well handled in the Alberta school system. I may be expecting too high a level of attainment, but I do not think so.

Results prove that the General Mathematics Classes are quite capable of learning the graphical concept. They probably require more time and different methods of instruction in order to achieve results comparable with other classes.

The idea of negative numbers is quite confusing to many students. I believe that the use of the graph is a good way to clear up this troublesome problem.

The method of analysis, as employed in this school, in teaching General Mathematics 1, gives good results. I used more time on this topic than is given in most schools; but I believe that more time should be given to each topic in General Mathematics classes than to corresponding classes composed of "A" and "B" grade students. "C" grade students require more time to master a problem than do the average students.

The "C" student has been forced to try to keep up with others of higher intelligence, and he has been compelled to go on to new problems before he has mastered the basic principles of the old. The result is that the whole structure

is insecure since it rests on a weak foundation.

If we take fewer topics and treat them more thoroughly, not simply more exhaustively, we will tend to restore the student's self confidence. Having acquired more self confidence, he will face the world with a much better chance of success.

I believe we should endeavor to make good followers rather than leaders of the "C" students. Notice the adjective "good". By this I mean intelligent followers, not blind followers. The students should be trained to see clearly what the problem is, even if they cannot solve it. As applied to graphs this means that they should be able to read them intelligently even if they do not learn how to construct them.

The topic of graphs is taken up in all grades from seven to twelve. I believe that the results of the tests prove that the graphical concept is not well enough understood even by senior high school pupils. I believe that the topic should be treated more fully at one point in the high school course, possibly at the grade nine level.

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